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Significance of Stress-Block Parameters on the Moment Capacity of Sections Under-Reinforced with FRP

by G. Urgessa, S. Horton, M.M. Reda Taha, and A. Maji

Synopsis: For under-reinforced concrete sections reinforced with FRP, failure of a member is initiated by rupture of the FRP bar and the typical ACI compression stress-block might not be applicable. This is because of the fact that the corresponding strain at the extreme fiber of the concrete will not reach the ultimate strain in concrete. Therefore, accurate computation of flexural capacity requires developing equivalent stress-block parameters that represent the stress distribution in the concrete at a particular strain level. While the ACI 440 permits the use of a simplified approach to calculate moment capacities that do not require equivalent stress-block calculations, the significance of this simplification needs to be examined. This paper suggests a family of curves based on the extreme fiber strain in concrete using three existing stress-strain models. The paper highlights the significance of these curves for different values of compressive strengths of concrete.

Keywords: FRP; HPC; stress-block parameters

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INTRODUCTION

The ACI 440 guide (ACI 440 2003) recommends the flexural capacity of an FRP reinforced concrete section to be computed with the same equations required by ACI 318 code requirements (ACI 318 2002).

$$\phi M_n \geq M_u \tag{1}$$

The nominal flexural strength is dependent on the controlling mode of failure. Failure can be initiated by concrete crushing or FRP rupture. The failure mode can be determined by comparing the FRP reinforcement ratio (ρ_f) to the balanced reinforcement ratio (ρ_{fb}) which can be calculated from Equations (2a) and (2b).

$$\rho_f = \frac{A_f}{bd} \tag{2a}$$

$$\rho_{fb} = 0.85\beta_1 \frac{f'_c}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}} \tag{2b}$$

If the reinforcement ratio is below the balanced ratio, FRP rupture failure mode governs. In this case, the typical compression stress-block might be no longer applicable because the ultimate strain in concrete, ε_{cu} = 0.003, might not be attained. While a simplified nominal moment capacity, M_n, can be computed from Equation (3), ACI 440 recommends a conservative calculation of the nominal flexural capacity, $M_{n\text{-}ACI}$, based on balanced section properties and a reduction factor of 0.8 as shown in Equation (4).

$$M_n = A_f f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right) \tag{3}$$

$$M_{n-ACI} = 0.8\, A_f f_{fu} \left(d - \frac{\beta_1 c_b}{2} \right) \tag{4}$$

More accurate results can be obtained by formulating an equivalent-stress block to approximate the stress distribution in the concrete at a particular strain level. This paper presents a family of curves for equivalent stress-block parameters using three stress-strain models. The analysis will include computation of concrete compressive strain at failure, the depth to the neutral axis and the equivalent stress-block parameters, α_{1E} and β_{1E}. In order to formulate equivalent stress-block parameters, different stress-strain models are investigated and the significance of the calculations on High Performance Concrete (HPC) is highlighted. The widespread use of HPC worldwide and the growing interest in producing concretes with high durability criteria (Aïtcin 1998) motivated our interest in examining the significance of combining HPC and FRP.

METHODS

Effective stress-block parameters

The generalized stress-block in concrete (Rüsch 1960) is non-linear from the extreme fiber to the neutral axis as shown in Figure 1. The parameters α and β are stress intensity and resultant location coefficients respectively. These coefficients can be computed from Equations (5a) and (5b).

$$\alpha = \frac{1}{f'_c} \int_0^c f_c(y)\, dy \tag{5a}$$

$$\beta = 1 - \frac{\int_0^c y.f_c(y)\, dy}{\int_0^c f_c(y)\, dy} \tag{5b}$$

The relationship between the generalized stress-block parameters (α and β) and the corresponding equivalent Whitney's rectangular stress-block parameters (α_{1E} and β_{1E}) can be developed through geometry and equating the resulting compressive force, C, from the two stress-blocks in Figure 1. The relationships are shown in Equations (6a) and (6b).

$$\alpha_{1E} = \frac{\alpha}{\beta_{1E}} \tag{6a}$$

$$\beta_{1E} = 2\beta \tag{6b}$$

Stress-strain models

In order to compute the effective stress-block parameters, there is a need to determine the stress-strain relationship in concrete, f_c and ε_c. A number of models describing the stress-strain curve of Normal Strength Concrete (NSC) were proposed through the past three decades. Popovics (1970) summarized some of the earlier empirical models for numerical approximations of stress-strain curves. Yip (1998) investigated various generic forms of equations for the constitutive stress-strain relationships of different grades of concrete. Relatively recently, a group of new models to describe the stress-strain curves of HPC have been developed (e.g. Loov 1991 and Collins et al. 1993). In this paper, three models are investigated.

Model 1 by Desayi and Krishnan (1964) -- Desayi and Krishnan (1964) proposed a simple model describing stress-strain of concrete as

$$\frac{f_c}{f'_c} = \frac{2(\varepsilon_c / \varepsilon'_c)}{1 + (\varepsilon_c / \varepsilon'_c)^2} \tag{7}$$

The model is simple in form such that closed-form integrations can be evaluated to calculate the stress-block parameters. The stress-strain curves resulting from this model are shown in Figure 2 for different strengths of concrete. The simplicity of the model has encouraged many researchers to use it as a general stress-strain model for concrete (MacGregor 1997).

It becomes obvious from Figure 2 that Desayi and Krishnan's model (1964) was developed to describe the stress-strain of normal strength concrete. The stress-strain curves for HPC as proposed by the model do not match those observed experimentally (Collins and et al. 1993; Macgregor 1997).

Model 2 by Loov (1991) -- Loov (1991) extended the early work by Carriera and Chu (1985), who proposed a model that can be adjusted to fit experimental values. The target was to formulate a generalized model capable of describing stress-strain of NSC and HPC.

$$\frac{f_c}{f'_c} = \frac{\varepsilon_c}{\varepsilon'_c} \frac{A}{1 + B(\varepsilon_c / \varepsilon'_c) + C(\varepsilon_c / \varepsilon'_c)^n} \tag{8a}$$

where, $C = \dfrac{1}{n-1}$ (8b)

$$A = 1 + B + C \tag{8c}$$

Loov (1991) had determined the values of n and B experimentally for different concrete strengths. In order to plot the stress-strain curve for any other specific strength of concrete, polynomial regression can be used to evaluate the corresponding values of *A*, *B,* *C* and *n*. The proposed stress-strain curves are shown in Figure 3.

Model 3 by Collins and et al. (1993) -- Collins, Mitchell and MacGregor (1993) extended the early work by Thorenfeldt and et al. (1987), who described the relationship between the compressive stress at any strain to the maximum stress. The descending part of the stress-strain curve is described well in the analysis through a post-stress decay factor. This model accounts for the fact that the stress-strain curves drop at a higher rate after the peak stress for HPC sections compared to NSC sections.

$$\frac{f_c}{f_c'} = \frac{\varepsilon_c}{\varepsilon_c'} \frac{n}{n-1+\left(\varepsilon_c / \varepsilon_c'\right)^{nk}} \tag{9a}$$

where, $n = 0.8 + \dfrac{f_c'}{17}$ [MPa units] (9b)

For the ascending part of the curve, $k = 1$, otherwise $k = 0.67 + \dfrac{f_c'}{62}$ [MPa units] (9c)

$$\varepsilon_c' = \frac{f_c'}{E_c} \frac{n}{n-1} \tag{9d}$$

$$E_c = 3320\sqrt{f_c'} + 6900 \quad \text{[MPa units]} \tag{9e}$$

The stress-strain curves representing Collins and et al. model (1993) is shown in Figure 4.

The ratio of resultant location and stress intensity coefficients

An estimate of the ultimate concrete compression strain based solely on the moment due to the compressive concrete stresses can be obtained by solving the differential equation

$$\frac{\partial M}{\partial\left(\varepsilon_c / \varepsilon_c'\right)} = 0 \tag{10a}$$

By examining the parameter involved in determining the moment capacity from generalized stress-block diagram (see Figure 1), the moment about the neutral axis due to tension in the FRP bars can be expressed as

$$M = A_f f_{fu}\left(d - \beta c\right) \tag{10b}$$

The depth to the neutral axis can be determined from equilibrium of the tension force in the FRP bars and the compression force of concrete.

$$c = \frac{A_f f_{fu}}{\alpha f_c' b} \tag{10c}$$

Combining Equations (10b) and (10c), the general expression for the moment about the neutral axis for a given FRP reinforced cross-section can be determined as a function of the ratio of β and α.

$$M = \frac{\left(A_f f_{fu}\right)^2}{f_c' b}\left(\frac{b\, d\, f_c'}{A_f f_{fu}} - \frac{\beta}{\alpha}\right) \tag{10d}$$

The nominal moment capacity, M_n, is thus obtained when the ratio of (β/α) is minimum.

$$M_n = \frac{\left(A_f f_{fu}\right)^2}{f_c' b}\left(\frac{b\, d\, f_c'}{A_f f_{fu}} - \left(\frac{\beta}{\alpha}\right)_{min}\right) \tag{10e}$$

RESULTS

A computer program using MATLAB® is developed to generate families of curves describing the generalized and equivalent stress-block parameters curves based on the three models described above. While closed form solutions can be developed using Desayi and Krishnan model (1964), numerical integration was implemented to the other two models. The curves are developed for concrete strengths ranging from 20 MPa to 100 MPa.

Generalized stress-block parameters

A plot of the generalized stress-block parameters, α and β, for each stress-strain model is generated using Equations (5a) and (5b). The families of curves are shown in Figures 5 to 10.

The ratio of resultant location and stress intensity coefficients

Based on equation (10e), the ultimate concrete compression strain in concrete corresponds to the minimum β/α ratio. Figures 11, 12 and 13 show the plot of the ratio β/α for the three stress-strain models. Values of concrete strain corresponding to the minimum (β/α) ratio are presented in Table 1.

Equivalent stress-block parameters for design of under-reinforced FRP sections

The equivalent stress-block parameters, α_{1E} and β_{1E}, are determined from equations (6a) and (6b) using the generalized stress-block parameters computed above. The equivalent stress-block parameters proposed for use of under-reinforced FRP sections are shown in Figures 14 to 19.

Comparison of nominal moment capacities

To compare between these models and to examine their significance given the ACI 440 recommendations (ACI 440 2003), moment capacity analysis was performed on a number of concrete sections under-reinforced with GFRP bars and different cross-section properties. A simplified form of Equation (10e) as shown in Equation 11 is used while implementing the equivalent stress-block parameters for the three described models with β_{1E} representing the value of β when the ratio of β to α is minimum. The predicted flexural capacities using Equations (3) and (4) are also presented based on balanced section parameters. GFRP material properties of f_{fu} = 617 MPa and E_f = 42 GPa are used. The results are presented in Table 2.

$$M_n = A_f f_{fu}\left(d - \frac{\beta_{1E} c}{2}\right) \tag{11}$$

DISCUSSIONS

Rigorous analysis of the moment capacities using the equivalent stress-block parameters and Equation (3) and (4) show the value $\beta_{1E}\,c$ (the depth of equivalent stress-block for FRP rupture) is less than $\beta_1\ c_b$ (the depth of equivalent stress-block for a balanced failure). Hence, the ACI 440 model underestimates the moment capacity compared with the moment capacities computed using the three models. If the reduction coefficient 0.8, used in the ACI model is eliminated from Equation (4), the ACI estimated moment capacity will be close but still less than the moment capacities obtained using the three models. However, such recommendation cannot be made until both the ACI and the equivalent stress-block moment capacities are compared to experimental results. This comparison is currently being investigated.

It can be observed that the Desayi and Krishnan model (1964) will result in a higher moment capacity (small values of β/α) compared to models 2 and 3. The two other models predicted values of moment capacity closer to each other than model 1. Observing the (β/α) curves in Figures 11 to 13, it can be argued that a limiting compressive strain value of 0.0035 for HPC might be more practical than the 0.003 value currently recommended by ACI. This observation confirms the findings by Loov (1991) and values of maximum strain obtained from specimens of Ibrahim and MacGregor (1997), which have been adopted by recent Canadian codes for using FRP in concrete structures (ISIS Canada 2001).

CONCLUSIONS

Families of curves representing equivalent stress-block parameters were generated for accurate moment capacity analysis of sections under-reinforced with FRP using three stress-strain models for HPC. Further investigation is underway to compare these findings to experimental results for concrete sections under-reinforced with FRP.

NOTATIONS

A , B , C , n	= curve fitting parameters for use in Loov model
A_f	= area of FRP reinforcement
b	= width of concrete cross-section
C	= concrete compressive stress
c	= distance from extreme compression fiber to the neutral axis
d	= distance from extreme fiber to centroid of tension reinforcement

E_c	= modulus of elasticity of concrete
E_f	= guaranteed modulus of elasticity of the FRP
f_c	= compressive stress in concrete
f'_c	= characteristic compressive strength of concrete
f_{fu}	= design tensile strength of FRP
k	= stress decay factor by Collins and et al. model
M	= moment due to compressive stresses
M_n	= nominal moment capacity
M_u	= factored moment at section
T	= tension force in FRP
y	= distance from the neutral axis to any location through the depth
α	= generalized stress intensity coefficient
α_{1E}	= equivalent rectangular stress intensity coefficient
β	= resultant location coefficient for a generalized stress-block
β_1	= ACI resultant location coefficient
β_{1E}	= resultant location coefficient for an equivalent stress-block
ε_c	= strain in concrete
ε'_c	= strain in concrete at maximum stress
ε_{cu}	= ultimate strain in concrete
ϕ	= strength reduction factor
ρ_f	= FRP reinforcement ratio
ρ_{fb}	= FRP reinforcement ratio producing balanced strain conditions

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Table 1— Minimum ratio of (β/α) and the corresponding strain values

f'_c [MPa]		20	40	60	80	100
Desayi and Krishnan model	$(\beta/\alpha)_{min}$	0.527	0.527	0.527	0.527	0.527
	ε_c	0.0026	0.0029	0.0033	0.0037	0.0041
Loov model	$(\beta/\alpha)_{min}$	0.532	0.542	0.556	0.567	0.581
	ε_c	0.0031	0.0029	0.0032	0.0036	0.0035
Collins and et al. model	$(\beta/\alpha)_{min}$	0.526	0.541	0.552	0.561	0.568
	ε_c	0.0027	0.0028	0.003	0.0033	0.0035

Table 2 — Moment Capacity Comparison

				Model 1	*Model 2*	*Model 3*	*Eqn* (3)*	*Eqn** (4)*
b	*d*	ρ_f	f'_c	*Nominal Moment Capacities*				
[mm]	[mm]	%	[MPa]	[kN·m]				
350	550	0.402	40	253	252.2	252.6	245.3	196.2
350	550	0.536	40	334.1	333.4	334.9	327.0	261.6
350	450	0.491	40	205.7	205.2	205.5	200.8	160.6
350	450	0.655	40	271	270.6	270.8	267.6	214.1
500	550	0.282	40	255	254.3	254.6	245.3	196.2
500	550	0.375	40	337.8	337	337.4	327.0	261.6
350	550	0.402	80	255.8	254.5	254.6	248.1	198.5
350	550	0.536	80	339.4	337.4	337.4	330.9	264.7
350	450	0.491	80	208.5	207.4	207.4	203.1	162.5
350	450	0.655	80	276.5	274.8	274.8	270.8	216.6
500	550	0.282	80	257	255.9	255.9	248.1	198.5
500	550	0.375	80	341.4	339.7	339.9	330.9	264.7

* Nominal moment capacity based on Equation (3)
** Nominal moment capacity based on ACI Equation (4)

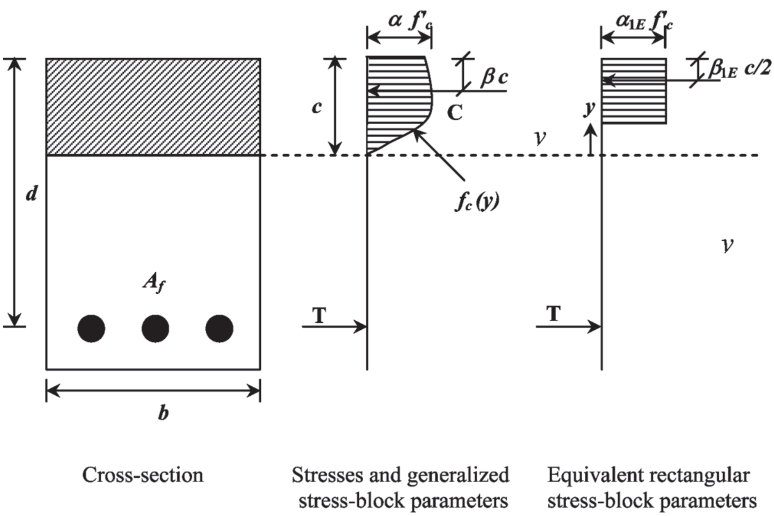

Figure 1 — Stress-block parameters for rectangular cross-section

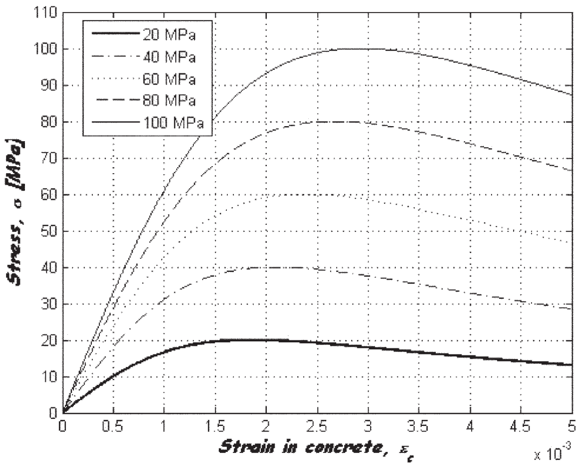

Figure 2 — Stress-strain relationship of concrete based on Desayi and Krishnan model (1964)

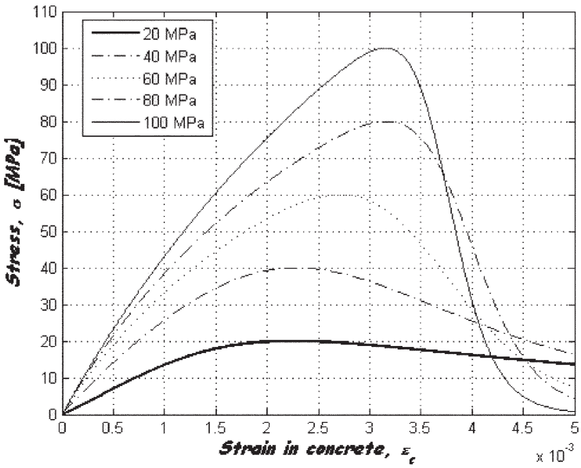

Figure 3 — Stress-strain relationship of concrete based on Loov model (1991)

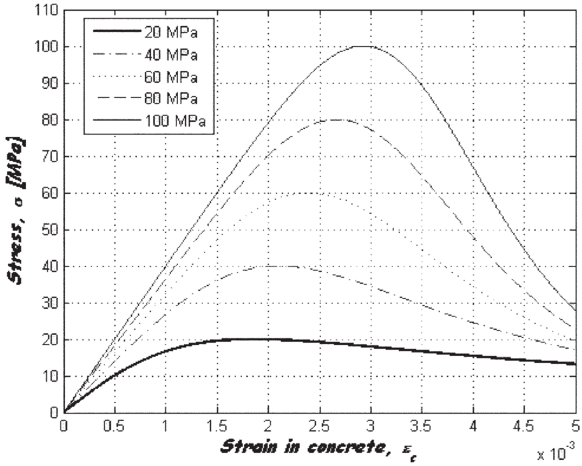

Figure 4 — Stress-strain relationship of concrete based on Collins et al. model (1993)

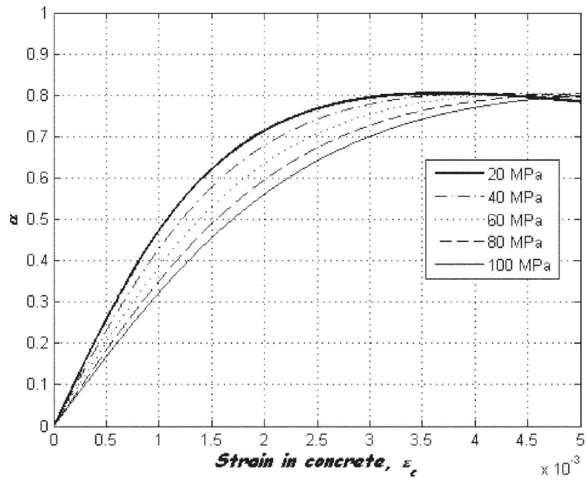

Figure 5 — Generalized stress-block parameter, α, using Desayi and Krishnan model (1964)

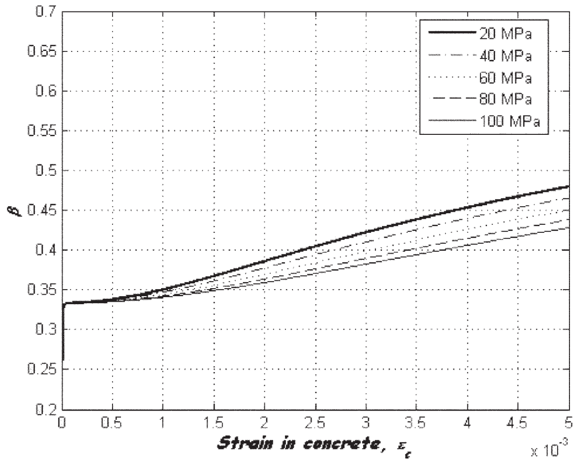

Figure 6 — Generalized stress-block parameter, β, using Desayi and Krishnan model (1964)

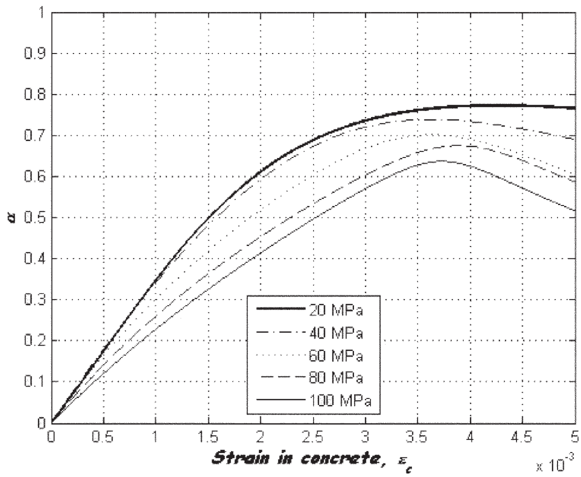

Figure 7 — Generalized stress-block parameter, α, using Loov model (1991)

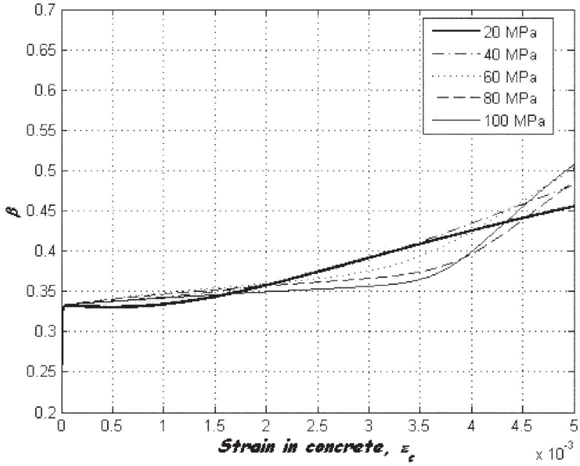

Figure 8 — Generalized stress-block parameter, β, using Loov model (1991)

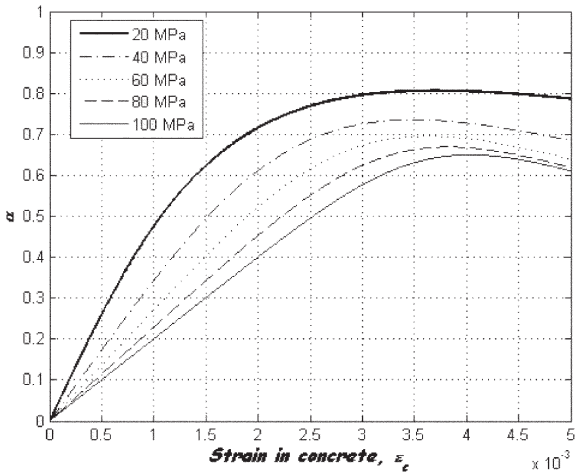

Figure 9 — Generalized stress-block parameter, α, using Collins et al. model (1993)

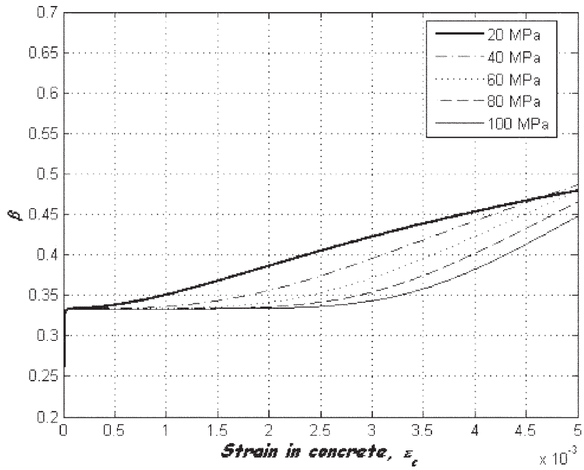

Figure 10 — Generalized stress-block parameter, β, using Collins et al. model (1993)

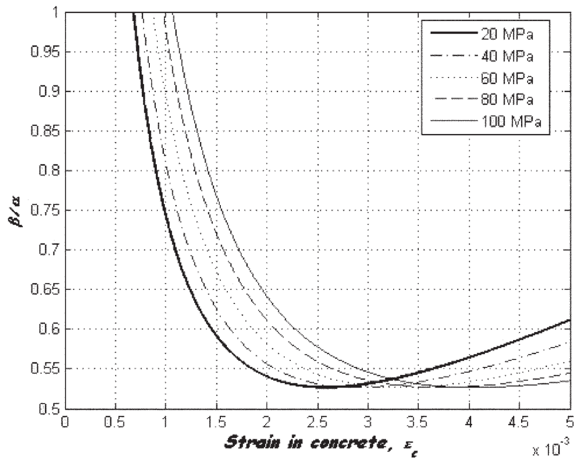

Figure 11— The ratio of β/α using Desayi and Krishnan model (1964)

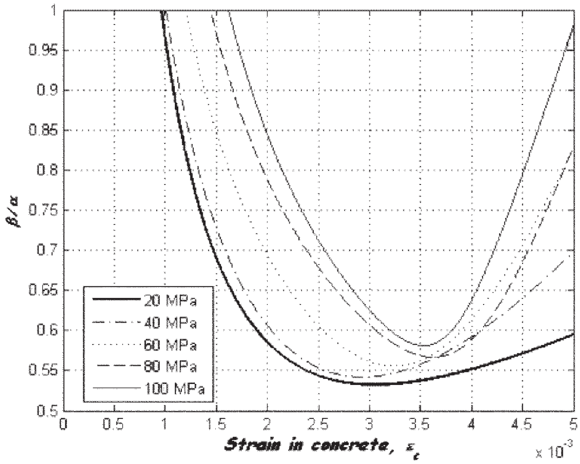

Figure 12 — The ratio of β/α using Loov model (1991)

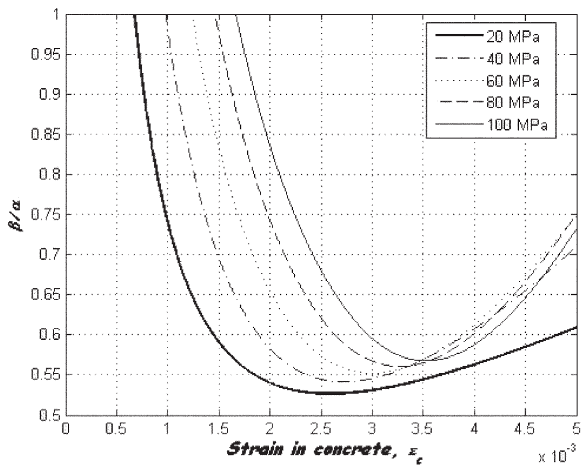

Figure 13 — The ratio of β/α using Collins et al. model (1993)

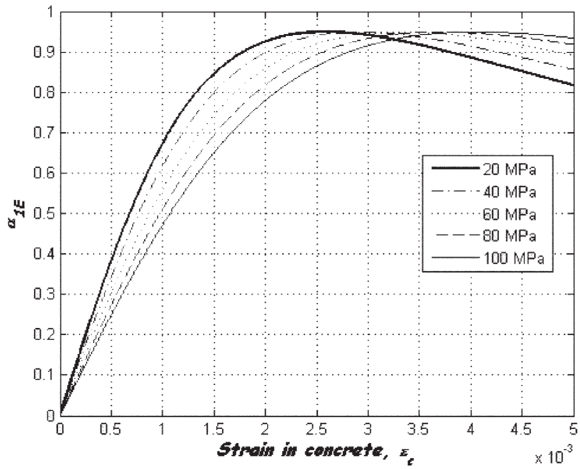

Figure 14 — Equivalent stress-block parameter, α_{1E}, using Desayi and Krishnan model (1964)

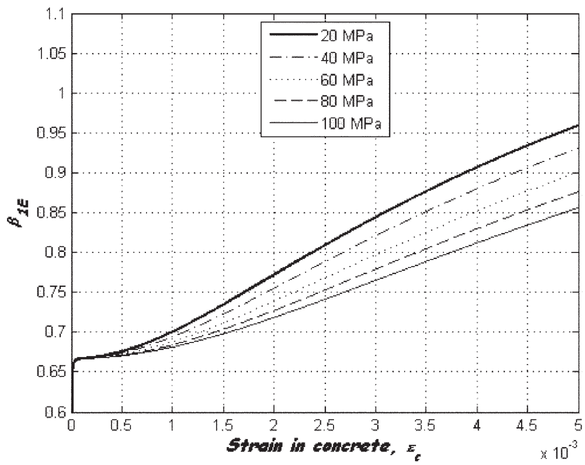

Figure 15 — Equivalent stress-block parameter, β_{1E}, using Desayi and Krishnan model (1964)

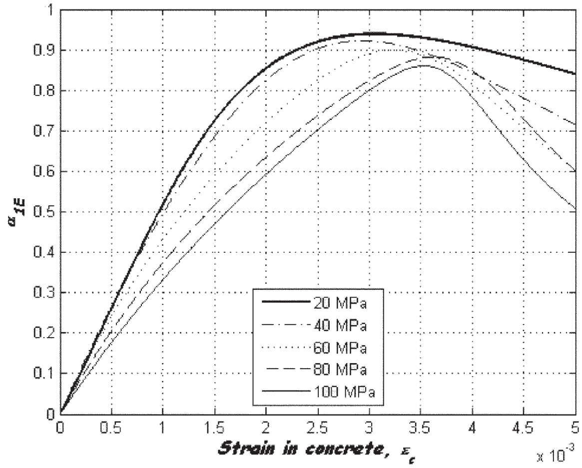

Figure 16 — Equivalent stress-block parameter, α_{1E}, using Loov model (1991)

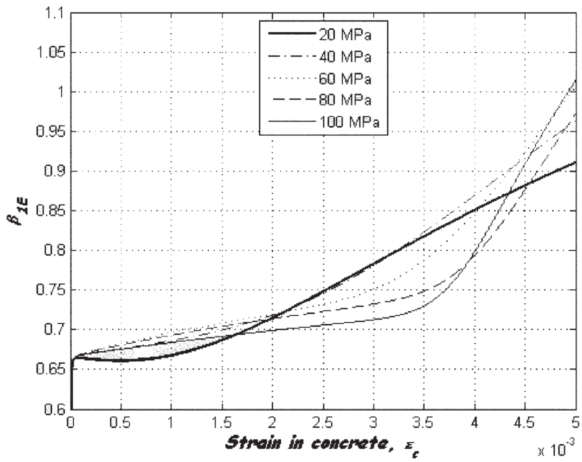

Figure 17 — Equivalent stress-block parameter, β_{1E}, using Loov model (1991)

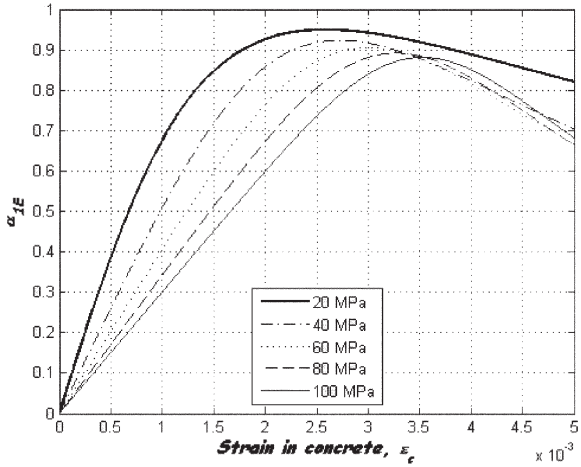

Figure 18 — Equivalent stress-block parameter, α_{1E}, using Collins et al. model (1993)

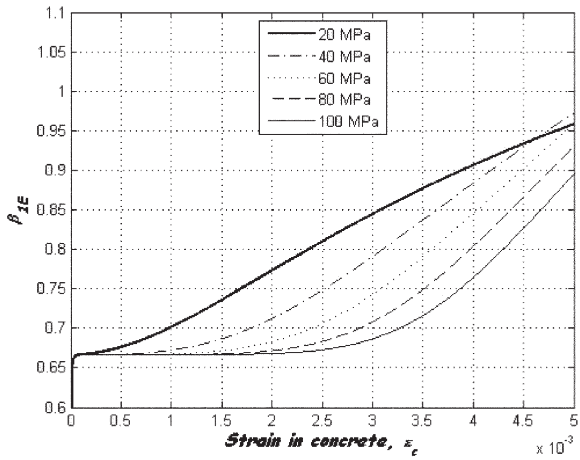

Figure 19 — Equivalent stress-block parameter, β_{1E}, using Collins et al. model (1993)