# Rationale for the ACI 440.1R-06 Indirect Deflection Control Design Provisions

by C.E. Ospina and S.P. Gross

Synopsis: Compared to ordinary steel reinforcement, Fiber-Reinforced Polymer (FRP) reinforcing bars have a lower stiffness, display a brittle-elastic response, and possess particular bond characteristics. The dependence on these distinctive features makes deflection control in FRP-reinforced concrete beams and one-way slabs a more elaborate process compared to the traditional serviceability design of steel-reinforced members. This paper reports the rationale and fundamental concepts backing the indirect deflection control procedure for concrete beams and one-way slabs reinforced with FRP bars adopted by ACI 440.1R-06. The fundamental procedure can be applied regardless of the type of reinforcement; it is independent of the member's stiffness through the cracked stage; and it is expressed as a function of the deflection-span ratio, which allows designers to fully control deflections depending on applicable serviceability limits. The paper also explains the simplifications made to the fundamental procedure that led to the development of the indirect deflection control procedure in tabular form found in ACI 440.1R-06, including the method by which tension stiffening effects are accounted for.

Keywords: deflection; FRP-reinforced members; indirect deflection control; one-way slabs; serviceability design; tension stiffening

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## INTRODUCTION

For serviceability design of FRP-reinforced members, ACI 440.1R-06 provides guidance on preliminary sizing of concrete beams and one-way slabs through an indirect deflection control procedure that is expressed in terms of minimum beam and one-way slab thickness requirements. The designer is then required to calculate deflections directly based on a modification of the traditional Branson's effective moment of inertia equation because, unlike in ACI 318-05, the indirect deflection control procedure in ACI 440.1R-06 is not intended to waive the direct deflection control calculation.

The procedure used here to develop the indirect deflection control provisions in ACI 440.1R-06 can be applied to any type of reinforcement. It is also independent of the member's stiffness, which is difficult to evaluate through the cracked stage, and it is directly related to a deflection-span ratio, which allows designers to control the serviceability design depending on applicable deflection limits. After presenting the general procedure, the paper **explains** the development of the minimum thickness table found in ACI 440.1R-06. The fundamental assumptions used in the simplification of the general procedure are identified, and the method by which the procedure is adjusted for tension stiffening effects is presented.

## BACKGROUND

Deflections of reinforced concrete beams and one-way slabs can be controlled directly or indirectly. Direct deflection control refers to the calculation of deflections and their comparison with allowable limits. Direct methods span from traditional elastic theory to advanced finite element analyses. ACI Committee 435 (1974) and Branson (1977) report comprehensive summaries of classic direct deflection control procedures for steel-reinforced concrete beams and flat plates. Indirect deflection control procedures limit deflections by determining maximum span-depth ratios, minimum depths, or minimum tension reinforcement ratios (ACI Committee 435, 1978) that satisfy a given deflection-span ratio, $\Delta_m/L$. The latter is defined by experience. In the context of steel-reinforced members, Branson (1977) recommends using indirect procedures for initial member proportioning and then checking deflections directly. Deflections can also be controlled by means of appropriate construction practice. Precambering and delaying removal of forms are some of the preferred options.

Deflection control provisions for steel-reinforced beams and one-way slabs in ACI 318-05 are concerned with deflections that occur at service levels due to immediate and sustained static loads. Effects from dynamic loads such as earthquakes, winds, or vibration of machinery are not considered. Two methods are given: i) the indirect method of controlling the minimum thickness of the member (ACI 318 Table 9.5a); and ii) the direct method of limiting computed deflections (ACI 318 Table 9.5b). The choice of method is left to the discretion of the designer. Table 1 shows values of minimum thickness for non-prestressed steel-reinforced beams and one-way slabs per ACI 318-05 expressed as maximum span-depth ratios. Table 2 shows the allowable deflections per ACI 318-05.

The direct deflection control design provisions for FRP-reinforced concrete beams and one-way slabs in ACI 440.1R-06 follow a format similar to that of ACI 318-05. Deflections are calculated directly using a modified version of Branson's effective moment of inertia equation, developed by Gao, Benmokrane and Masmoudi (1998). Based on recent data published by Yost, Gross, and Dinehart (2003) and other researchers, the equation used in earlier versions of ACI 440.1 was modified by the second author and adopted by ACI Committee 440 for inclusion in ACI 440.1R-06.

At its Washington D.C. 2004 Spring meeting, ACI Sub-committee 440H commissioned the authors to develop an indirect deflection control table similar to Table 9.5(a) of ACI 318 to define maximum span-depth ratios for FRP-reinforced concrete beams and one-way slabs, based on the indirect deflection control approach proposed by Ospina, Alexander, and Cheng (2001). The goal of the sub-committee was to provide designers guidance for preliminary sizing of members in the form of typical span-depth ratios required to satisfy serviceability design criteria. The indirect deflection control table developed by the authors, which may be found in Appendix A, was approved by ACI 440 in its San Francisco 2004 Fall meeting. Presentation of the rationale behind the development of this table constitutes the main subject of this paper.

## PROPOSED INDIRECT DEFLECTION CONTROL PROCEDURE

As reported by Branson (1977), the instantaneous midspan deflection, $\Delta_m$, of a reinforced concrete beam or one-way slab subjected to a uniformly distributed load can be calculated as

$$\Delta_m = K_1 \left(\frac{5}{48}\right)\left(\frac{M_m L^2}{E_c I_e}\right) \tag{1}$$

where $M_m$ is the midspan bending moment, $L$ is the span length, $E_c$ is concrete's elastic modulus, and $I_e$ is an effective moment of inertia. The constant $K_1$ depends only on boundary conditions, and is defined as

$$K_1 = 1.2 - 0.2 \frac{M_o}{M_m} \tag{2}$$

where $M_o$ is the statical moment, i.e. $M_o = \frac{wL^2}{8}$, $K_1 = 1$ for simply supported spans, $K_1 = 0.8$ for fixed-hinged beams, and $K_1 = 0.6$ for fixed-fixed beams. For cantilevered spans, $K_1 = 2.4$, $M_m$ is replaced with the moment at the support, and $\Delta_m$ is replaced with the deflection at the free end. In Eq. 2, both $M_o$ and $M_m$ result from the same loading.

Due to the difficulty in evaluating $I_e$ to account for cracking effects on flexural stiffness, Eq. 1 can be rewritten independent of $I_e$ (Ospina, Alexander and Cheng 2001) as

$$\Delta_m = K_1 \left( \frac{5}{48} \right) \psi_m \, L^2 \tag{3}$$

where $\psi_m$ is the curvature at midspan. Assuming cracked-elastic behavior,

$$\psi_m = \frac{\varepsilon_{rm}}{d(1 - k_m)} \tag{4}$$

where $\varepsilon_{rm}$ is the reinforcement strain at midspan (or support for a cantilevered span), $d$ is the effective flexural depth, and $k_m$ is the ratio of the neutral axis depth to the flexural depth, also at midspan (or support for a cantilevered span), calculated as

$$k_m = \sqrt{(n\rho_r)^2 + 2n\rho_r} - n\rho_r \tag{5}$$

where $n$ is the modular ratio, $n = E_r / E_c$, and $\rho_r$ is the reinforcement ratio. Dividing both sides of Eq. 3 by $L$ and substituting Eq. 4 into 3 leads to

$$\frac{\Delta_m}{L} = K_1 \left( \frac{5}{48} \left( \frac{\varepsilon_{rm}}{d(1 - k_m)} \right) \right) L \tag{6}$$

Rearranging terms and setting $\eta = d/h$, the maximum span-depth ratio is given by

$$\frac{L}{h} \leq \frac{48\eta}{5\,K_1} \left( \frac{1 - k_m}{\varepsilon_{rm}} \right) \frac{\Delta_m}{L} \tag{7}$$

where $\eta$ may be assumed to vary from 0.85 to 0.95.

In Eq. 7, both the reinforcement strain and the neutral axis location define a limiting curvature that is consistent with an allowable deflection-span ratio. The interdependence of the reinforcement strain, the deflection-span ratio and the span-depth ratio is further illustrated in Fig. 1, which shows the central portion (segment ABCD) of a beam with midspan deflection $\Delta_m$ and curvature $\psi_m$. If the deflection, and hence curvature, is to remain unchanged, an increase in the service strain in the reinforcement from $\varepsilon_{rm}$ to $\varepsilon'_{rm}$ must be accompanied by deepening the beam (segment ABC'D'), reducing the span-depth ratio, as implied by Eq. 7.

The merits of Eq. 7 are evident: it can be applied to members reinforced with either ordinary steel or FRP bars; it is independent of the member's flexural stiffness; and the direct dependency on the deflection-span ratio allows the designer to control the serviceability design based upon specific deflection limits. Table 3 shows a list of allowable deflection-span limits for structural, sensorial and aesthetic reasons, adapted from ACI 435.3R-68 (ACI Committee 435 1968), which can be used in conjunction with Eq. 7, depending on the specific application. A very comprehensive list of allowable deflection-span limits can be found in Branson (1977).

## VALIDATION OF PROPOSED MODEL

Figure 2 shows the effect of the reinforcement strain level at midspan, $\varepsilon_{rm}$, and different span fixity conditions on the maximum span-depth ratio assuming $\Delta_m/L$ = 1/240, $k_m$ = 0.195, $\rho_r E_r$ = 96 ksi (661.9 MPa), $\eta$ = 0.9, and $f'_c$ = 5 ksi (34.5 MPa). Values correspond to a one-way slab with a reinforcement ratio that is about 3.5 times the balanced reinforcement ratio prescribed by ACI 440.1R-06, thereby implying a compressive failure at ultimate. The variation in the span support conditions is represented by the $M_o/M_m$ ratio. For instance, a value of $M_o/M_m$ of about 2.0 simulates an edge span supported by a masonry wall at the edge, with the first interior support continuous. In a prototype interior span, $M_o/M_m$ varies from about 2.8 to 3.0. In a simple span, $M_o/M_m = 1$.

Figure 2 shows how the maximum span-depth value decreases as $\varepsilon_{rm}$ increases, as was observed conceptually in Fig. 1. Figure 2 also shows that the effect of boundary conditions on the span-depth ratio is more noticeable at lower reinforcement strain levels.

To validate the proposed indirect deflection control procedure, let us consider a one-way slab with ordinary steel reinforcement, assuming $\varepsilon_{rm}$ = 0.0012. This strain value is roughly 60% of the yield strain of steel, which is often considered a target strain level in steel-reinforced concrete serviceability design. Assuming simple support conditions, Fig. 2 shows that $\varepsilon_{rm}$ = 0.0012 leads to a span-depth ratio of 24.2. This value is close to the lower bound span-depth ratio of 20 prescribed by ACI 318-05 for simply-supported steel-reinforced one-way slabs. The latter is somewhat lower, because, as pointed out by Branson (1977), the span-depth ratios in ACI 318 include several modifying factors for different conditions, including long-term effects. Generally speaking, this comparison confirms that using a deflection-span ratio, $\Delta_m/L$, equal to 1/240, where $\Delta_m$ is calculated on the basis of an instantaneous deflection under total service load, results in span-depth ratios that are relatively consistent with those given in ACI 318-05.

Assume now that the slab is reinforced with GFRP bars and that $\varepsilon_{rm}$ is equal to 0.002. This FRP strain value is 5/3 greater than 0.0012, which is consistent with ACI 440.1R-06 rationale of allowing larger crack widths in members with FRP due to FRP's superior corrosion resistance. For $\varepsilon_{rm}$ = 0.002 and assuming simple support conditions, Fig. 2 renders a span-depth ratio of 14.5 which means that the GFRP-reinforced slab needs be about 1.7 times thicker than the steel-reinforced slab, for the same span length. The increased depth is the result of allowing a higher reinforcement strain at service level. Note that if the target GFRP strain were 0.0012, the corresponding span-depth requirement would be close to 24.2. Evidently, since the elastic modulus of GFRP is lower than steel's, say $E_r$ = 6,000 ksi (41,370 MPa), the GFRP-reinforced slab would have a reinforcement ratio that is 29,000/6,000 = 4.83 times that of the steel-reinforced slab. It is worth noting, however, that this comparison is rather simplistic because bond between GFRP bars and concrete differs from that between steel bars and concrete, which implies that the resulting span-depth limit may deviate from 24.2.

For comparable span lengths, the higher member depth requirement associated with FRP-reinforced beams and one-way slabs relative to their steel-reinforced counterparts has significant economic consequences. These translate directly into higher costs due to greater concrete volumes and high FRP reinforcement ratios. Thus far, however, the proposed indirect deflection control formulation has neglected the effect of tension stiffening (i.e. the tensile contribution of concrete between cracks) on deflection. It can be hypothesized that span-depth requirements in FRP-reinforced members may be relaxed if this effect is introduced in the indirect deflection control procedure. It is worth noting that the indirect deflection control table in ACI 318-05 does not explicitly account for tension stiffening effects.

## TENSION STIFFENING EFFECT ON INDIRECT DEFLECTION CONTROL

To account for concrete's tension stiffening effect on the proposed indirect deflection control procedure, it is necessary to express the span-depth requirements as a function of the average curvature instead of the curvature at a crack. The procedure (Ospina, Alexander, and Cheng 2001) makes use of the tension stiffening model given by CEB/FIB MC90 (1990). Accordingly, Eq. 1 can be rewritten as

$$\frac{\Delta_m}{L} = \frac{5}{48} K_1 L \left[(1-\xi)\,\psi_1 + \xi\,\psi_2\right] \tag{8}$$

For members with rectangular cross-section, the midspan curvatures $\psi_1$ (at uncracked section level) and $\psi_2$ (at fully cracked level) are defined as

$$\psi_1 = \frac{2 f_r}{E_c\, h} \tag{9}$$

$$\psi_2 = \frac{\varepsilon_{rm}}{d\left(1-k_m\right)} \tag{10}$$

and

$$\xi = 1 - \beta_1 \beta_2 \left( \frac{M_{cr}}{M_m} \right)^2 \quad \geq 0.4 \tag{11}$$

where $M_{cr}$ is the cracking moment and the coefficients $\beta_1$ and $\beta_2$ characterize, respectively, the bond quality of the bars ($\beta_1$ = 1.0 for high bond bars) and the influence of load duration or repetition ($\beta_2$ = 1.0 for first loading). For FRP-reinforced concrete members subjected to short-term first loading, Hall (2000) recommends $\beta_1\beta_2 = 0.5$.

Substituting Eqs. 9 and 10 into Eq. 8, assuming $f_r = 7.5\sqrt{f_c'}$ ($f'_c$ in psi), $E_c = 57000\sqrt{f_c'}$ ($f'_c$ in psi), $\eta = d/h$, and rearranging terms, leads to

$$\frac{L}{h} \leq \frac{48\eta}{5K_1}\left[\frac{1}{\left[(1-\xi)\dfrac{15\eta}{57000} + \xi\dfrac{\varepsilon_{rm}}{(1-k_m)}\right]}\right]\frac{\Delta_m}{L} \tag{12}$$

Figure 3 illustrates the tension stiffening effect on the maximum span-depth ratio prediction for the simply-supported slab of the validation example, for different service level-to-cracking moment ratios, $M_m / M_{cr}$, assuming $\beta_1\beta_2 = 0.5$. Equation 12 leads to a curve that approaches asymptotically a span-depth ratio of 14.5, which is that in absence of tension stiffening effects. The figure shows that concrete's tensile contribution significantly affects the maximum span-depth requirement, especially at load levels that are slightly greater than the cracking load; within these load levels, tension stiffening leads to an increased span-depth ratio, i.e. the member deepening requirement is relaxed. At higher load levels, the tension stiffening effect attenuates due to bond degradation.

## INDIRECT DEFLECTION CONTROL PROVISIONS IN ACI 440.1R-06

Taking into account the inherent difficulties associated with the standardization of Eq. 12, some fundamental assumptions and simplifications were required to develop an indirect deflection control table for FRP-reinforced beams and one-way slabs. The procedure is described in this section. The table showing the recommended minimum thicknesses for the design of beams and one-way slabs with FRP bars is shown in Table 4.

The starting point was to establish that the table would be developed using Eq. 7, which not only requires less parameters to be known than does Eq. 8 but also allows for a simple tension stiffening correction: the one used in the direct deflection control provisions of ACI 440.1R-06. The authors felt that, as critical as it was to consider tension stiffening, it was just as important to consider an equation that was consistent with other provisions of the ACI 440.1R-06 document (recall that Eq. 12 is based on the CEB/FIP MC90 tension stiffening model).

The use of Eq. 7 in design is not straight forward because neither $k_m$ nor $\varepsilon_{rm}$ are known prior to a detailed analysis. For this reason, it was decided to conduct a parametric analysis based on typical conditions for FRP-reinforced concrete flexural members in order to determine both $k_m$ and $\varepsilon_{rm}$. In the analyses, an arbitrary deflection-span limit of 1/240 (i.e. a maximum instantaneous deflection of *L*/240) was assumed under total service load. This value was not chosen to endorse a deflection limitation of *L*/240, but rather because of the relative consistency it provides with the span-depth limitations suggested in ACI 318-05, as indicated by the results presented in Fig. 2. For simplicity, explicit modifications for time-dependent behavior and other factors were not applied in deriving the ACI 440.1R-06 table; however, such modifications can implicitly be addressed by adjusting the limiting deflection-span ratio to account for assumed time-dependent deflection multipliers and ratios of dead load to live load. The assumed deflection-span ratio is clearly stated in the text of ACI 440.1R-06 to provide the designer guidance when applying the table. Since the table is only intended for preliminary member sizing, the designer is permitted to adjust a suggested minimum thickness based on less restrictive or more restrictive deflection-span limits.

Limiting span-depth ratios were first computed in the absence of tension stiffening effects, i.e. according to Eq. 7. Then, the tension stiffening effect was accounted for by multiplying the resulting span-depth ratios by the ratio of the effective moment of inertia to the cracked section moment of inertia, $I_e/I_{cr}$, where $I_e$ is calculated using the modified Branson's equation for FRP, given in ACI 440.1R-06 as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_d I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \tag{13}$$

where

$$\beta_d = \frac{1}{5}\left(\frac{\rho_f}{\rho_{bf}}\right) \le 1 \tag{14}$$

In Eq. 14, $\rho_f$ and $\rho_{bf}$ are, respectively, the reinforcement ratio and the balanced reinforcement ratio for an FRP-reinforced member.

A description of the calculations performed in the analyses, including all fundamental assumptions, are provided in Table 5, which facilitates interpretation of Table 6. The latter shows the results of a typical analysis, in this case for a simply-supported one-way slab with 5 ksi (34.5 MPa) concrete and GFRP reinforcement. Numerous other tables were generated but were omitted due to space limitations.

Table 7 provides an overall summary of the analyses conducted, and forms the basis for the development of Table 4. Sixteen basic cases were considered, resulting from four different support conditions, two different reinforcement types (GFRP and CFRP), and two different member types (beam and slab). For each case, computations were performed considering four different reinforcement quantities, equivalent to 1.0, 2.0, 3.0, and 4.0 times the balanced reinforcement ratio, respectively. For one-way slabs and beams, the assumed ratio of service moment to calculated nominal moment was taken as 0.3 and 0.4, respectively. This difference is intended to reflect the fact that slabs, on a relative basis, tend to be more lightly loaded. Reinforcement properties for GFRP and CFRP reinforcement were assumed to represent typical values for commercially available bars. As can be seen in Table 7, the type of FRP reinforcement does not have a significant effect on the computed span-depth ratios. For this reason, the number of basic cases considered reduces to eight, with each case corresponding to a value in the final version of the ACI 440.1R-06 minimum thickness table.

Values from Table 7 chosen for inclusion in ACI 440.1R-06 Table 8.2 are indicated in italics. Span-depth ratios for one-way slabs are based on the analyses of sections reinforced at 2.0 times the balanced reinforcement ratio whereas those for beams correspond to analyses of sections reinforced at 3.0 times the balanced reinforcement ratio. These choices reflect the general differences in reinforcing levels for beams and slabs. While it can be argued that lower reinforcement ratios are reasonable for designs in many cases, the data shows no significant difference in the computed values between, for example, the cases of 1.0 and 2.0 times the balanced reinforcement ratio.

## CONCLUSIONS AND RECOMMENDATIONS

Defining a maximum span-depth ratio as a vehicle for indirect control of deflections in concrete beams and one-way slabs reinforced with FRP bars is affected by FRP's stiffness, brittle-elastic nature, and bond properties. To overcome the limitations imposed by the influence of many intervening variables on the deflection calculations, a general indirect deflection control model for FRP-reinforced beams and one-way slabs is proposed. The model can be applied to a wide variety of support conditions regardless of the type of reinforcement. The procedure is also independent of the member's effective moment of inertia, which is difficult to quantify across the cracked stage, and it is expressed in terms of an allowable deflection-span ratio, which allows designers to have full control of the serviceability design depending on applicable deflection limits.

According to the proposed method, the maximum span-depth ratio in concrete beams or one-way slabs with FRP reinforcement is particularly affected by the level of FRP strain at a crack at service load level. For comparable span lengths, concrete beams and one-way slabs with FRP need be deepened to satisfy the same maximum deflection-span ratios for steel-reinforced members. If concrete's tension stiffening effect is accounted for, the member deepening penalty can be relaxed, especially at load levels that are roughly greater than that at first flexural cracking.

A series of indirect deflection control parametric analyses were performed on beams and one-way slabs reinforced with FRP bars based on the proposed model. Concrete's tension stiffening effect was accounted for through some simplifications. Based on these results, the maximum span-depth ratios reported in Table 4 are proposed. This table was adopted by ACI 440 for inclusion in ACI 440.1R-06. The table is intended only for use in the preliminary sizing of members, and does not supersede the requirement for designers to check deflections directly, as stipulated by ACI 440.1R-06.

Experimental evidence studying deflections in FRP-reinforced concrete beams and one-way slabs under uniformly distributed gravity loads for different support conditions is needed to further examine the quality of the proposed deflection control procedures.

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## LIST OF SYMBOLS

| | |
|---|---|
| $d$ | Effective flexural slab or beam depth |
| $E_c$ | Elastic modulus of concrete, ksi (MPa) |
| $E_f$ | Elastic modulus of FRP bars, ksi (MPa) |
| $f_r$ | Concrete's modulus of rupture, ksi (MPa) |
| $f'_c$ | Specified cylinder compressive strength of concrete, ksi (MPa) |
| $h$ | Slab thickness or beam depth |
| $I_{cr}$ | Cracked moment of inertia |
| $I_e$ | Effective moment of inertia |
| $k_m$ | Ratio of neutral axis-to-flexural depth at midspan, for cracked-elastic conditions |
| $K_1$ | Constant depending on boundary conditions |
| $L$ ,$\ell$ | Span length |
| $M_a$ | Applied moment |
| $M_{cr}$ | Cracking moment |
| $M_m$ | Midspan moment |
| $M_o$ | Statical moment |
| $n$ | Modular ratio |
| $\beta_1$ | Bond coefficient in CEB/FIP MC90 |
| $\beta_2$ | Performance coefficient in CEB/FIP MC90 |
| $\Delta_m$ | Midspan deflection |
| $\varepsilon_{rm}$ | Reinforcement strain at midspan |
| $\rho_{bf}$ | Balanced reinforcement ratio for FRP reinforced member in ACI 440.1R-06 |
| $\rho_f$ | Reinforcement ratio for FRP reinforced member in ACI 440.1R-06 |
| $\rho_r$ | Reinforcement ratio |
| $\psi_m$ | Midspan curvature |
| $\psi_1$ | Curvature at uncracked section level in CEB/FIP MC90 |
| $\psi_2$ | Curvature at fully cracked section level in CEB/FIP MC90 |
| $\xi$ | Tension stiffening factor in CEB/FIP MC 90 |

**Appendix A -- ACI 440.1R-06 Indirect Deflection Control Design Provisions**

8.3.2.1 *Recommended minimum thicknesses for design*—Recommended minimum thicknesses for design of one-way slabs and beams are provided in Table 8.2. The table is only intended to provide guidance for initial design, and use of these recommended minimum thicknesses does not guarantee that all deflection considerations will be satisfied for a particular project.

Table 8.2 – Recommended minimum thickness of nonprestressed beams or one-way slabs

| | Minimum Thickness, *h* | | | |
|---|---|---|---|---|
| | Simply-supported | One end continuous | Both ends continuous | Cantilever |
| Solid one-way slabs | $\ell/13$ | $\ell/17$ | $\ell/22$ | $\ell/5.5$ |
| Beams | $\ell/10$ | $\ell/12$ | $\ell/16$ | $\ell/4$ |

Values in Table 8.2 are based on a generic maximum span-to-depth ratio limitation (Ospina, Alexander, and Cheng 2001) corresponding to the limiting curvature associated with a target deflection-span ratio (Eq. 8-10). The procedure can be applied for any type of reinforcement.

$$\frac{\ell}{h} \leq \frac{48\eta}{5K_1}\left(\frac{1-k}{\varepsilon_f}\right)\left(\frac{\Delta}{\ell}\right)_{\max} \qquad (8\text{-}10)$$

In Eq. (8-10), $\eta = d/h$, $k$ is as defined in Eq. (8-12), and $(\Delta/\ell)_{max}$ is the limiting service load deflection-span ratio. $K_1$ is a parameter that accounts for boundary conditions. It may be taken as 1.0, 0.8, 0.6, and 2.4 for uniformly loaded simply-supported, one-end continuous, both ends continuous, and cantilevered spans, respectively. The term $\varepsilon_f$ is the strain in the FRP reinforcement under service loads, evaluated at midspan except for cantilevered spans. For cantilevers, $\varepsilon_f$ shall be evaluated at the support.

Eq. (8-10) assumes no tension stiffening. To consider the effects of tension stiffening in developing Table 8.2, the values resulting from Eq (8-10) were modified by the ratio of effective and fully cracked moments of inertia, computed using Eq. (8-13a) and (8-11), respectively. Tabulated values are based on an assumed service deflection limit of $l/240$ under total service load, and assumed reinforcement ratios of $2.0\rho_{bf}$ and $3.0\rho_{bf}$ for slabs and beams, respectively.

**Table 1 -- Minimum Thickness of Nonprestressed Beams or One-way Slabs Unless Deflections are Computed (ACI 318-05 Table 9.5(a))**

| | Minimum Thickness, *h* | | | |
|---|---|---|---|---|
| | **Simply supported** | **One end continuous** | **Both ends continuous** | **Cantilever** |
| Member | **Members not supporting or attached to partitions or other construction likely to be damaged by large deflections** | | | |
| Solid one-way slabs | $\ell/20$ | $\ell/24$ | $\ell/28$ | $\ell/10$ |
| Beams or ribbed one-way slabs | $\ell/16$ | $\ell/18.5$ | $\ell/21$ | $\ell/8$ |

Note: Span length $\ell$ in inches

**Table 2 -- Maximum Permissible Computed Deflections (ACI 318-05 Table 9.5(b))**

| Type of Member | Deflection to be considered | Deflection limitation |
|---|---|---|
| Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections | Immediate deflection due to live load | $l/180$ |
| Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections | Immediate deflection due to live load | $l/360$ |
| Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections | That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) | $l/480$ |
| Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections | | $l/240$ |

**Table 3 -- Miscellaneous Deflection Limitations (Adapted from ACI 435 1968)**

| Reasons for limiting deflections | Examples | Deflection Limitation* | Portion of total deflection on which the deflection limitation is based |
|---|---|---|---|
| **Sensory Acceptability** | | | |
| Visual | Droopy cantilevers and sag in long span beams | By personal reference | Total deflection |
| Tactile | Vibrations of floors that can be felt<br><br>Lateral building vibrations | L/360<br><br>No recommendation | Full live load<br><br>Gust portion of wind |
| Auditory | Vibrations producing audible noise | Not permitted | |
| **Serviceability of Structure** | | | |
| Surfaces which should drain water | Roofs, outdoor decks | L/240 | Total deflection |
| Floors which should remain plane | Gymnasia and bowling alleys | L/360 + camber or<br><br>L/600 | Total deflection<br><br>Incremental deflections after floor is installed |
| Members supporting sensitive equipment | Printing presses and certain building mechanical equipment | Manufacturer's recommendations | Incremental deflections after equipment is leveled |
| **Effect on nonstructural elements** | | | |
| Walls | Masonry and plaster | L/600 or 0.3 in. (7.6 mm) max. or $\phi = 0.00167$ rad. | Incremental deflections after walls are constructed |
| | Metal movable partitions and other temporary partitions | L/240 or 1 in. (25.4 mm) max. | Incremental deflections after walls are constructed |
| | Lateral building movement | 0.15 in. (3.8 mm) offset per story<br>0.002 x height | Five min. sustained wind load |
| | Vertical thermal movements | L/300 or 0.60 in. (15.2 mm) max. | Full temperature differential movement |
| Ceilings | Plaster<br>Unit ceilings such as accoustic tile | L/360<br>L/180 | Incremental deflections after ceiling is built |
| Adjacent building elements supported by other members | Windows, walls and folding partitions on unyielding supports below the deflecting member | Absolute deflection limited by tolerances built into the element in question | Incremental deflection after building element in question is constructed |
| **Reasons for limiting deflections** | **Examples** | **Deflection Limitation*** | **Portion of total deflection on which the deflection limitation is based** |
| **Effect on structural elements** | | | |
| Deflections causing instability of primary structure | Arches and shells<br>Long columns | Effect of deflections on the stresses and stability of the structure should be taken into account in the design | |
| Deflections causing different force system or change in stresses in some other element | Beam bearing rotation on masonry wall | Effect of deflections on the stresses and stability of the structure should be taken into account in the design | |
| Deflections causing dynamic effects | Resonant vibrations which increase static deflections and stresses such as those produced by wind, dancing, moving loads and machinery | Dynamic deflections should be added to static deflections and the total should be less than the limitations imposed for other reasons | |

* Deflection limitations are given for members supported at both ends and for cantilevers, except as noted. It is assumed that the supports do not move.

**Table 4 -- Recommended Minimum Thickness of Nonprestressed Beams or One-way Slabs Reinforced with FRP Bars**

| | Minimum Thickness, *h* | | | |
|---|---|---|---|---|
| | Simply-supported | One end continuous | Both ends continuous | Cantilever |
| Solid one-way slabs | $\ell/13$ | $\ell/17$ | $\ell/22$ | $\ell/5.5$ |
| Beams | $\ell/10$ | $\ell/12$ | $\ell/16$ | $\ell/4$ |

**Table 5 -- Description of Parametric Analysis Calculations**

| Row in Table 6 | Description |
|---|---|
| 1 to 5 | Specify material properties. A concrete strength of 5 ksi (34.5 MPa) was used for all analyses presented here. However, it should be noted that the analyses were not very sensitive to the use of higher strength concrete. The use of 10 ksi concrete (69 MPa), as opposed to 5 ksi, generally increased the calculated minimum thicknesses by about 10 to 20%. |
| 6 | Identifies the $K_1$ factor per selected span support conditions. Values of 1.0, 0.8, 0.6, and 2.4 were used for simply-supported, one-end continuous, both ends continuous, and cantilevered spans, respectively. |
| 7 | Refers to the assumed *d/h* ratio, taken as 0.90 for all analyses. |
| 8 | Identifies the allowable immediate deflection under total service load, assumed to be *L*/240 for all analyses. |
| 9 | Identifies the assumed ratio between service and nominal moments. This ratio is used to establish the service moment after calculating the nominal moment directly. For slabs and beams, this ratio was taken as 0.30 and 0.40, respectively, to reflect the fact that slabs tend to be more lightly reinforced. |
| 10 to 12 | Identify the balanced reinforcement ratio and the reinforcement ratios assumed for the analyses (1.0, 2.0, 3.0, and 4.0 times the balanced ratio) |
| 13 | Computes the normalized neutral axis depth, $k_m$ |
| 14 to 16 | Respectively, compute the reinforcement stress at ultimate, the reinforcement stress at service, and the reinforcement strain at service ($\varepsilon_{rm}$ in Eq. 7) |
| 17 to 19 | Respectively, compute normalized cracking, service, and nominal moments |
| 20 to 22 | Respectively, compute normalized gross, cracked, and effective moments of inertia. The effective moment of inertia is calculated using Eq. 13. |
| 23 and 24 | Used in the computation of the effective moment of inertia (column 22) |
| 25 | Quantifies the tension stiffening effect as a multiplier, in terms of the ratio between the effective and cracked moments of inertia |
| 26 | Computes the recommended span-depth ratio (without consideration of tension stiffening) per Eq. 7. |
| 27 | Computes the recommended span-depth ratio based on Eq. 7, with a correction made to account for tension stiffening effects (Col. 27 = Col. 26 x Col. 25) |

Table 6 -- Maximum Span-Depth Ratios for Simply-Supported Slab ($f'_c$ = 5 ksi and GFRP Reinforcement)

| Row | Description | Parameter | Unit | Values | | | |
|---|---|---|---|---|---|---|---|
| 1 | Material Properties | $f_{fu}$ | ksi | 100 | | | |
| 2 | | $E_f$ | ksi | 6000 | | | |
| 3 | | $f'_c$ | ksi | 5 | | | |
| 4 | | $\beta_1$ | | 0.80 | | | |
| 5 | | $E_c$ | ksi | 4031 | | | |
| 6 | Assumptions | $K_1$ | | 1.00 | | | |
| 7 | | $\eta$ | | 0.90 | | | |
| 8 | | $L/\Delta_m$ | | 240 | | | |
| 9 | | $M_s/M_n$ | | 0.30 | | | |
| 10 | Reinforcement Ratios | $\rho_{bf}$ | | 0.0052 | | | |
| 11 | | $\rho_f/\rho_{bf}$ | | **1.00** | **2.00** | **3.00** | **4.00** |
| 12 | | $\rho_f$ | | 0.0052 | 0.0104 | 0.0156 | 0.0207 |
| 13 | Neutral Axis | $k_m$ | | 0.117 | 0.161 | 0.193 | 0.220 |
| 14 | Reinforcement Stresses/Strains | $f_{f,ult}$ | ksi | 100.00 | 68.34 | 54.36 | 46.05 |
| 15 | | $f_{rm,serv}$ | ksi | 29.30 | 19.85 | 15.69 | 13.23 |
| 16 | | $\varepsilon_{rm,serv}$ | | 0.0049 | 0.0033 | 0.0026 | 0.0022 |
| 17 | Normalized Moments | $M_{cr}/bd^2$ | ksi | 0.109 | 0.109 | 0.109 | 0.109 |
| 18 | | $M_s/bd^2$ | ksi | 0.146 | 0.195 | 0.228 | 0.254 |
| 19 | | $M_n/bd^2$ | ksi | 0.487 | 0.650 | 0.761 | 0.848 |
| 20 | Normalized Moments of Inertia | $I_g/bd^3$ | | 0.1143 | 0.1143 | 0.1143 | 0.1143 |
| 21 | | $I_{cr}/bd^3$ | | 0.0066 | 0.0123 | 0.0175 | 0.0223 |
| 22 | | $I_e/bd^3$ | | 0.0134 | 0.0181 | 0.0231 | 0.0278 |
| 23 | Tension Stiffening Effect | $M_s/M_{cr}$ | | 1.34 | 1.79 | 2.09 | 2.33 |
| 24 | | $\beta_d$ | | 0.200 | 0.400 | 0.600 | 0.800 |
| 25 | | $I_e/I_{cr}$ | | 2.04 | 1.48 | 1.32 | 1.24 |
| 26 | Recommended Span to Depth Ratios | $(L/h)_{NO\ TS}$ | | 6.5 | 9.1 | 11.1 | 12.7 |
| 27 | | $(L/h)_{w/\ TS}$ | | **13.3** | **13.5** | **14.6** | **15.9** |

**Table 7 -- Summary of Maximum Span-Depth Ratio Computations**

| Case | | | $f_{fu}$ | $E_f$ | $f'_c$ | $K_1$ | $\eta$ | $L/\Delta_m$ | $M_s/M_n$ | $(L/h)_{w/TS}$ at $\rho_f/\rho_{bf}$ = | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ksi | ksi | ksi | | | | | 1.00 | 2.00 | 3.00 | 4.00 |
| Slab | Simply-Supported | GFRP | 100 | 6000 | 5 | 1.0 | 0.90 | 240 | 0.30 | 13.3 | *13.5* | 14.6 | 15.9 |
| Slab | Simply-Supported | CFRP | 300 | 20000 | 5 | 1.0 | 0.90 | 240 | 0.30 | 11.6 | *13.0* | 14.7 | 16.2 |
| Beam | Simply-Supported | GFRP | 100 | 6000 | 5 | 1.0 | 0.90 | 240 | 0.40 | 7.0 | 8.2 | *9.4* | 10.5 |
| Beam | Simply-Supported | CFRP | 300 | 20000 | 5 | 1.0 | 0.90 | 240 | 0.40 | 6.8 | 8.5 | *9.9* | 11.2 |
| Slab | One End Continuous | GFRP | 100 | 6000 | 5 | 0.8 | 0.90 | 240 | 0.30 | 16.6 | *16.9* | 18.3 | 19.8 |
| Slab | One End Continuous | CFRP | 300 | 20000 | 5 | 0.8 | 0.90 | 240 | 0.30 | 14.5 | *16.3* | 18.3 | 20.2 |
| Beam | One End Continuous | GFRP | 100 | 6000 | 5 | 0.8 | 0.90 | 240 | 0.40 | 8.8 | 10.3 | *11.8* | 13.2 |
| Beam | One End Continuous | CFRP | 300 | 20000 | 5 | 0.8 | 0.90 | 240 | 0.40 | 8.5 | 10.6 | *12.4* | 14.0 |
| Slab | Both Ends Continuous | GFRP | 100 | 6000 | 5 | 0.6 | 0.90 | 240 | 0.30 | 22.1 | *22.5* | 24.4 | 26.4 |
| Slab | Both Ends Continuous | CFRP | 300 | 20000 | 5 | 0.6 | 0.90 | 240 | 0.30 | 19.4 | *21.7* | 24.4 | 27.0 |
| Beam | Both Ends Continuous | GFRP | 100 | 6000 | 5 | 0.6 | 0.90 | 240 | 0.40 | 11.7 | 13.7 | *15.7* | 17.6 |
| Beam | Both Ends Continuous | CFRP | 300 | 20000 | 5 | 0.6 | 0.90 | 240 | 0.40 | 11.3 | 14.1 | *16.5* | 18.6 |
| Slab | Cantilevered | GFRP | 100 | 6000 | 5 | 2.4 | 0.90 | 240 | 0.30 | 5.5 | *5.6* | 6.1 | 6.6 |
| Slab | Cantilevered | CFRP | 300 | 20000 | 5 | 2.4 | 0.90 | 240 | 0.30 | 4.8 | *5.4* | 6.1 | 6.7 |
| Beam | Cantilevered | GFRP | 100 | 6000 | 5 | 2.4 | 0.90 | 240 | 0.40 | 2.9 | 3.4 | *3.9* | 4.4 |
| Beam | Cantilevered | CFRP | 300 | 20000 | 5 | 2.4 | 0.90 | 240 | 0.40 | 2.8 | 3.5 | *4.1* | 4.7 |

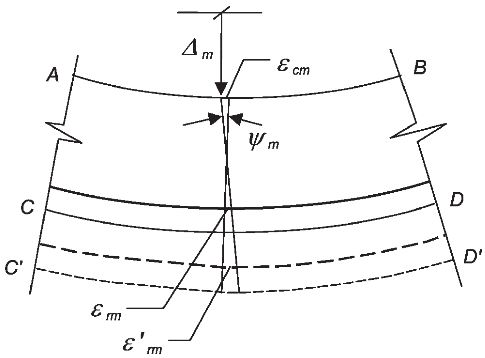


Figure 1 — Effect of Reinforcement Strain on Span-Depth Ratio

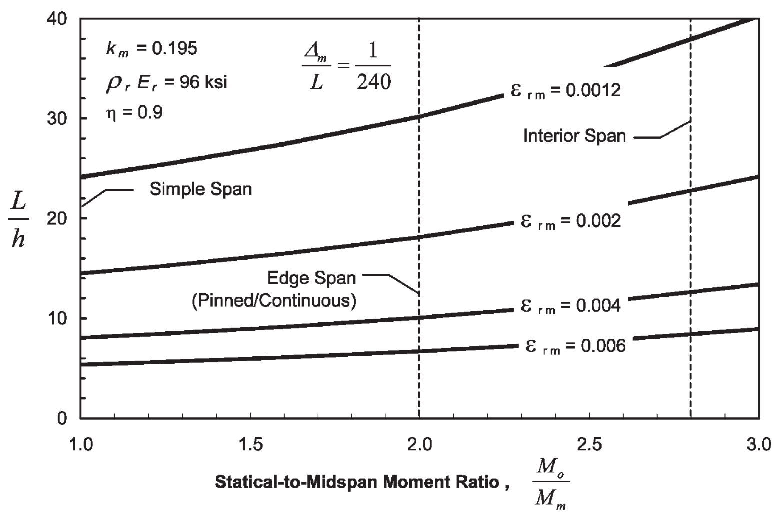


Figure 2 — Effect of Reinforcement Strain on Span-Depth Ratio

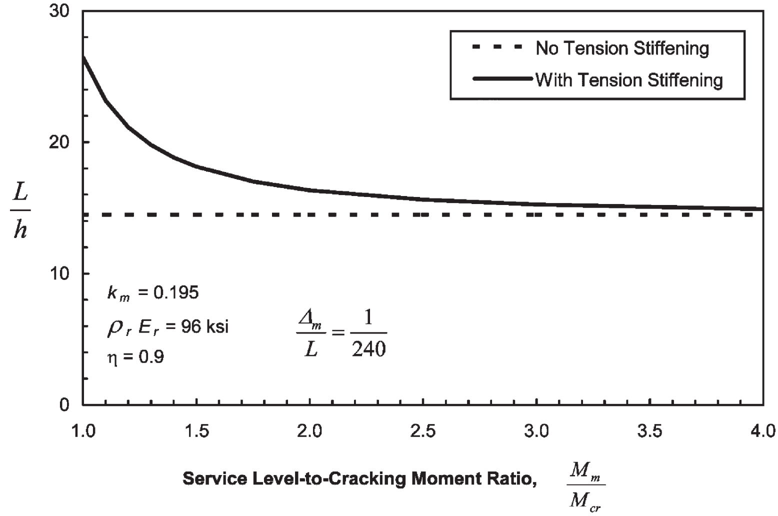


Figure 3 — Tension Stiffening Effect on Span-Depth Ratio