SP-230—42

Probabilistic Based Design for FRP Strengthening of Reinforced Concrete

by R.A. Atadero and V.M. Karbhari

Synopsis: This paper presents an approach for considering a wide variety of composite materials in reliability-based design of composite strengthening of concrete structures. The approach is based on the use of the mean value of composite properties as the design value and having a composite specific resistance factor that is a function of the COV of the composite properties. Strengthening design of a simple beam with three model composites is used to demonstrate the feasibility of this approach. For comparison, designs are created using two existing strengthening guidelines, ACI 440 and TR55. These designs are shown to have a high degree of variability in their reliability and demonstrate the need for a reliability based design procedure. Some of the critical areas of further research required for further development of this design procedure are briefly discussed.

Keywords: design guidelines; material design values; reliability-based design; strengthening

Rebecca A. Atadero is a Ph.D. student in the Department of Structural Engineering at the University of California, San Diego.

Vistasp M. Karbhari is a professor in the Department of Structural Engineering and the Material Science Program at the University of California, San Diego.

INTRODUCTION

Fiber reinforced polymer (FRP) composites are proving to be a valuable material in the retrofit and repair of reinforced concrete structures. As such, a number of guidelines for their use have been developed. [1, 2, 3, 4] These guidelines all contain provisions for topics of major concern in design of FRP strengthening, such as long-term performance and debonding. These guidelines are also similar in that though the design equations are presented in the partial factor formats that are often used in probability-based design, they are not true probabilistic codes. Instead, they typically make use of already existing design factors for loads and non-FRP materials, which in some cases are reliability based, and then provide additional factors for the FRP contribution. The factors for FRP are based on the judgment of the guideline-authoring agency based on available data. As these guidelines are not specifically calibrated from a reliability standpoint they suffer from the variation in reliability that is often associated with older design procedures. To remedy this problem it is important to get the probabilistic code development process underway.

The earliest attempt at reliability based design of composite strengthening was conducted by Plevris et. al. in 1995 [5]. Their study of flexurally strengthened beams examined the sensitivity of reliability to changes in the design variables and calibrated a set of resistance factors for use in design. Okeil et. al.[6] also examined the reliability of designs for flexural strengthening in their analysis of bridge girders. Their study used several girder designs and assumed different levels of steel degradation. Again resistance factors were calibrated based on the assumed design parameters. A common limitation of these studies is that the design factors were calibrated with one or two particular composite systems in mind. In the study by Okeil et. al. an analytically derived coefficient of variation of 2.2 percent was used for the composite failure strain. Tests of field-manufactured panels[7] suggest that this value is far too low to accurately represent wet layup composites, which are commonly used in the field due to the ease of application. Thus design factors for composite strengthening cannot be taken directly from these studies, but require further consideration of the materials involved.

The nature of composite materials provides many obstacles to the development of a reliability based design guideline. There are many different types of fiber and resin systems available, and they may be combined to produce finished composites of widely varying material properties. The variation in composite material properties is not only due to different constitutive materials; even two composites composed of the exact same constituent materials can show large differences in performance as a result of the manufacture and curing processes, particularly in the case of field manufactured wet layup.

One approach to this problem is the development of so-called “standard” composites, a set of composite materials that possess certain minimum properties. This, however, could be a shortsighted approach. Restricting strengthening design to certain established systems could hurt innovation in the materials. This approach also eliminates one of the prime features of composites, their tailorability. Finally, with so many systems currently in use how would a few standard composites be chosen? Thus the need is exposed for an approach that is broadly applicable, but which still satisfies the goals of reliability based design. This paper describes a proposed approach for creating a flexible, yet accurate, reliability based methodology for design of composite strengthening. An example of how the proposed code could be calibrated is presented, and compared to reliability results from designs based on other guidelines. Finally, some of the further research required in this field is outlined.

PROPOSED APPROACH FOR ACHIEVING BROAD APPLICABILITY OF RELIABILITY BASED DESIGN OF COMPOSITE MATERIALS

Resistance Factor as a Function of Composite Coefficient of Variation

The common approach to reliability-based design in the United States is Load and Resistance Factor Design (LRFD). In this design methodology factors are used to (generally) increase the expected load effects and decrease the predicted structural resistance. These factors are calibrated through reliability methods in an attempt to produce designs of a uniform, acceptably small, probability of failure over a range of design scenarios. For the case where the load and resistance are independent random variables, the probability of failure may be calculated using the integral shown in Equation 1, where $F_R(s)$ is the cumulative distribution function of the resistance and $f_S(s)$ is the probability distribution of the load.

$$p_f = \int_0^{\infty} F_R(s) f_S(s) ds \tag{1}$$

Equation 1 clearly shows that changes in the probability distribution of either load or resistance will cause changes in the probability of structural failure. As statistical parameters of load and resistance change, it is therefore necessary to change the design factors in order to reach the same target reliability. This integral does not represent the area of overlap between the distributions of load and resistance; however, the effect of changes in these distributions on the probability of failure can be qualitatively understood by considering the area of overlap. The size and shape of the overlapping region is largely a result of the spread in the distributions, and thus when they are calibrated the design factors are largely dependent on the amount of spread in the design variables. The degree of spread is typically described by the non-dimensional coefficient of variation (COV), defined as the standard deviation divided by the mean.

In design of FRP strengthening, the probability distribution of load for a particular member is not subject to change, however the distribution of resistance will be affected by statistical properties of the FRP. For composite materials, the COV of different material properties is very changeable. It may depend on the materials used to fabricate the composite, or, more often, on the way in which the composite was manufactured. Prefabricated strips are often characterized by COVs in the low single digits, while wet layup composites with poor quality control can show variation up to approximately 20%. Clearly in order for a design code to be applicable to a broad range of composites it must be able to accommodate a wide range of COVs.

Since a change in the composite COV changes the distribution of resistance, and thus the probability of failure, in order to maintain uniform reliability different resistance factors are need for different values of the COV. The design method could assume values of COV for certain types of materials, and then provide resistance factors for the different materials. However, a more straightforward approach is to express the resistance factors directly as a function of the COV. This is the approach taken by Alqam et al.[8] for probabilistic based design of FRP compression members.

Traditionally, the resistance factor, ϕ, in LRFD is used to account for all sources of uncertainty in the resistance. This includes variation in material properties; however, it also includes for the error that is present in the models used for design and possibly geometric variations. The traditional design equation is expressed as:

$$\sum \gamma_i Q_i \le \phi R \tag{2}$$

In this equation, γ_i denotes the load factors specific to the type of load, Qi represents the load effect due to different types of load (dead, live, snow, wind, etc.), ϕ is the resistance factor specific to the limit state being considered, and R is the nominal member resistance with respect to the limit state in question. For use of this design equation, there would need to be a different function of composite COV for ϕ for each limit state.

In another approach, two resistance factors could be used. The traditional ϕ would be used to account for variations in the steel and concrete, as well as the design model, while a separate factor, ψ, would be specific to the COV of the composite, and would account for the variation due to the composite. This is a break from traditional LRFD implementation, but is similar in some ways to the partial factor formats used in other parts of the world such as Europe and Canada. In this case the design equation would be expressed as:

$$\sum \gamma_i Q_i \le \phi R(\ldots, \psi x_{FRP}) \tag{3}$$

Here, ψ is a resistance factor specific the COV of the controlling FRP property, which is applied only to the FRP contribution to resistance. ϕ is a resistance factor specific to the limit state that considers the modeling inaccuracies, geometric variations, and variations in materials other than FRP. This approach is appealing because it isolates the effect of composite variability, making the effects of improved quality control very clear.

It would be ideal for the FRP dependent factor, ψ, to be constant for a given COV no matter what limit state is in question, however, different strengthening applications may be sensitive to variation in different composite properties. For example, for flexural strengthening, the design is usually governed by the ultimate strain of the composite, making the design quite sensitive to variation in ultimate stress. Other designs may be stiffness controlled, making the design most sensitive to modulus variation. For each type of design, the COV of the most significant composite property would be used to determine the resistance factor.

Use of Mean Value as Composite Design Allowable

The second feature of the proposed design approach for composite strengthening is to use the mean value of the composite properties as the design value. This is not a change from current practice with regard to the modulus and thickness; however, it is a very new approach with regard to strength. Currently, for most materials, the design strength is specified as a certain lower percentile of the test results, with the exact percentile varying depending on the material and the design specification. This is the approach taken by the guidelines that currently exist for FRP strengthening. By selecting the design value as a certain percentile of test results, the probability that the composite strength will fall below that value is fixed. However, the reliability is determined by the interaction of load and resistance and thus the probability of failure is not fixed by selecting a lower percentile for the design strength.

In a recent paper [9], the authors investigate the reliability implications of the current procedures used to define the material allowable for composite strength. The design procedures in ACI 440[1] are used to design strengthening for a sample beam. It is found that not only does the current design allowable not produce uniform levels of reliability as the COV of the composite strength is allowed to vary from 0.10 to 0.20, but in fact the designs for the COV equal to 0.20 are more reliable than the designs with the COV equal to 0.10. The higher reliability of the design created for the higher value of COV is explained in terms of the reliability integral in Equation 1. For this example, in order to have designs of uniform reliability, the material with a COV of 0.10 would require a smaller resistance factor than the material with a COV of 0.20. Clearly, this is an undesirable result. It implies that a composite that would generally be considered inferior due to the high variation is actually better. This effect is only an artifact of the total design procedure, which has conservatism built in with regard to the material allowable. However, if this design allowable is used for a probability-based code, designers will see that less demanding values of the resistance factor are used for materials with higher COVs. Since designers may not be familiar with the full reliability background, this could have unintended consequences such as giving no incentive for using higher quality materials, or lax quality control standards.

Another issue exposed by Atadero and Karbhari[9] and affecting the current design allowable is that designs where a higher percentage of the load is carried by FRP are more reliable than designs with smaller amounts of required FRP. This can be attributed to the small design values that often result when selecting a certain percentile of the strength distribution. These small design values result in bias factors, (ratio of mean to design value) that range up to 1.82 for a COV of 0.15 and 2.5 for a COV of 0.20 based on the material allowable definition in ACI 440[1]. For comparison, reinforcing steel has a bias factor of approximately 1.1. Based on these factors, it is obvious that when using current guidelines for designing FRP to carry load, much more conservatism is introduced than when using steel. This conservatism with regard to the FRP properties begins to assert itself as the amount of strengthening is increased. As more FRP is applied to carry a higher portion of the load, more reserve capacity is built into the structure due to these high bias factors. In their investigation of a slab strengthened to account for assumed levels of steel deficiency, Atadero and Karbhari[9] found that the reliability index increased from 3.16 for assumed 10 percent steel degradation to 4.50 for assumed 40 percent degradation.

These exposed difficulties with the current approach to material allowable specification could be handled with a complicated set of additional factors in the design procedure. However by using the mean value as the design value, the bias factor for the composite would be equal to 1, preventing the build-up of excess capacity as the percentage of load carried by the FRP increases. Furthermore, the inconsistency where higher COVs result in higher reliabilities would also be eliminated because the design value would no longer be dependent on the COV. This would eliminate the need for additional factors, resulting in a design method that is simpler to apply and one where even engineers who are not experts in reliability can understand the basis of the design approach.

Other advantages also exist to using the mean. For one, it is a value that is easy to obtain from manufacturer's data. The use of the mean also facilitates the use of additional design factors that can account for composite properties that may differ from test results. For example, a composite system for use in a rehabilitation project may be tested as a one-layer laminate, however during the design process it may be found that two layers are necessary to provide the required strength. Composites of different number of layers typically possess some differences in their properties, rather than requiring testing of the two-layer laminate a factor could be used to relate the mean of the one-layer property to the mean of the two-layer property. This idea could also be extended to account for specifics of manufacturing, such as worker experience or unusual curing conditions, that aren't generally considered when determining properties for design. It could even be used to reduce the design properties for the long-term degradation expected in service.

DESCRIPTION OF SAMPLE CALIBRATION

A simple example is presented to show how the features of the proposed approach could work. A generic beam is selected and strengthened with three different model composites, using the mean value as the design value for the composite properties. Different amounts of strengthening are applied depending on the composite resistance factor, which is allowed to range from 0.95 to 0.5 in increments of 0.05. The reliability of each design is assessed using Monte Carlo Simulation. Several different cases are evaluated, two different loading scenarios are used and for each scenario the reliability is evaluated at strength COVs of 0.05, 0.10, 0.15, and 0.20.

Sample Beam

The beam chosen for this demonstration is taken from Example 14.3 of ACI 440[1]. It is a simply supported beam with a length of 7.32 m (24 ft). The depth and width are 0.61 m (24 in) and 0.305 m (12 in), respectively, with a 0.546 m (21.5 in) depth to the reinforcing steel. The concrete compressive strength is assumed to be 34.5 MPa (5.0 ksi). The beam is reinforced with 3 ϕ 28 bars (No.9) and the reinforcement has a yield strength of 414 MPa (60 ksi). Strengthening is designed for the beam in anticipation of an assumed increase in live load of 50 percent using standard ACI load factors for dead and live load. The beam is sufficient in shear, and thus only flexural strengthening is required. The existing and anticipated moment effects on the beam are shown in Table 1.

Properties of Model Composites

Model carbon fiber reinforced composites are developed to represent three different levels of strength. All are assumed to have a rupture strain of 1.2 percent, thus the stronger composites are also stiffer. The thickness is assumed to be two-layers for all three cases. The assumed mean values for the properties for the three materials are shown in Table 2. As per the proposed approach, these values are used as the design values.

Design of Strengthening

The strengthening is designed following the method outlined in ACI 440. This method involves basic sectional analysis assuming a linear strain profile through the beam. The failure strain of the concrete is taken as 0.003. A bond-dependent coefficient, κ_m, is used to define a limit on the strain developed in the FRP to prevent the beam from failing due to composite debonding. The bond-dependent coefficient is a function of the composite modulus, thickness and rupture strain. The factor, κ_m, is multiplied by the rupture strain of the FRP to determine the limiting strain. The maximum value it may take is 0.9. For most designs, the composite strain limitation is the controlling criteria. Since the concrete is not crushing, the common stress block factors do not accurately describe the force carried in the concrete. In order to more accurately assess the force in the concrete, a parabolic stress distribution is assumed, and the stress block factors are computed using the equations found in Collins and Mitchell[10].

For this example, the format of the LRFD equation expressed in Equation 3 is used, with two factors ϕ and ψ. The value of ϕ is assumed fixed for this example at 0.9, a common resistance factor for the flexural failure mode. The value of ψ is varied from 0.95 to 0.5 in increments of 0.05. At each value of ψ the amount of composite needed to satisfy the design equation is determined. As all the composites are assumed to be two-layer laminates, the amount of material is controlled by adjusting the width of the composite strip. The needed width is computed in increments of 6.35 mm (0.25 in) so as to be as close as reasonable to the minimum required amount. The strengthening designs are summarized by the required composite width in Table 3. There is a significant decrease in the required amount of composite when switching from Material 1 to Material 2, however a similar decrease does not occur between Material 2 and Material 3. This is due to the high stiffness of Material 3, which results in a value of κ_m that is significantly smaller than the value used for Materials 1 and 2. With the strain of the composite severely limited by the low value of κ_m, nearly the same amount of Material 3 is needed as Material 2. For some smaller values of ψ it is not possible to satisfy the design equation.

Random Variables

Statistical descriptions of the design variables are necessary to analyze the reliability of the strengthening designs. For this analysis the geometric characteristics of the problem are considered deterministic, while the material properties and loads are allowed to vary.

Description of FRP Properties – The ultimate strength, modulus and thickness of the composite are considered as random variables. The strength is modeled with a Weibull distribution, a common choice for modeling composite strength. Modulus and thickness are modeled as lognormal distributions based on unpublished analysis of several FRP data sets. All distributions are fit using the method of moments, whereby the distribution parameters are determined by equating the assumed mean and standard deviation to the mean and standard deviation of the chosen distribution form. These variables are modeled as independent variables, assuming a complete lack of correlation. It is likely that there is indeed some correlation between the composite design variables, however without better data, assuming appropriate amounts of correlation is impossible. All of the statistical descriptions used here are assumptions based on limited actual data. As reliability based design of composite strengthening progresses, a key area of study will be statistical descriptions of the material properties.

The means of the composite properties are given in Table 2. Four different values are assumed for the COV of the strength; 0.05, 0.10, 0.15 and 0.20; to show how the reliability of a single design changes as the COV of strength is changed, and also to determine a relation between COV and the resistance factor, ψ. The COV of the modulus is taken as 12 percent and the COV of thickness is assumed to be 7 percent. The distribution parameters for the composite properties based on these values of mean and COV are presented in Table 4.

Description of Other Materials – The statistical description for steel is modeled on previous reliability studies. The yield strength of the steel is modeled as a lognormal variable with a bias factor of 1.1 and a coefficient of variation of 0.1 [11]. The modulus of steel is considered to be deterministic with a value of 200 GPa (29,000 ksi). The compressive strength of concrete is modeled as a normal variable with a bias factor of 1.14 [12] and a coefficient of variation of 0.15 [13]. This coefficient of variation corresponds to a normal level of quality control.

Description of Loads – Two different descriptions of loading are considered for reliability analysis. The first is appropriate for the design of new structures, with load statistics found in [14]. Dead load is modeled as a normal variable with a bias factor 1.05 and coefficient of variation 0.10. The live load is modeled as an extreme value type I distribution of maxima, also known as the Gumbel distribution. The live load has a bias factor of 1 and a coefficient of variation of 0.25.

The second loading scenario is the same as case 1, except the live load has a coefficient of variation of only 5 percent. This value is very low and is intended to represent the increased knowledge of a structure and the load demands placed on it at the evaluation and strengthening stage. The higher value of COV in the previous load case is appropriate for new design when the load demands are much more uncertain, and because conservatism at the original design stage adds very little to the cost of a structure. However, at the strengthening stage conservatism can be costly and thus it is often worth the extra effort to more accurately characterize the loads. Since the current load factors were not calibrated for such low levels of variation they are not applicable. Furthermore, it was desired to see the impact of the change in load variation on the same strengthening designs. However, since the variation is so low the designs had very high reliability. To lower the reliability into the typical range, the mean live load was increased from 173.6 kN-m (130 kip-ft) to 280.4 kN-m (210 kip-ft).

Reliability Analysis

Monte Carlo Simulation is used to determine the probability of failure. This is a procedure where random values of the design variables are selected from the appropriate distributions and are used to predict the resistance of the section. This resistance is then compared to load values randomly selected from the distributions describing loading. If the computed resistance is less than the randomly selected loads, the trial is considered a failure. This procedure is repeated a large number of times, and the probability of failure is estimated as the number of trials which failed divided by the total number of trials. For ease of comparison, this probability of failure is then related to the reliability index, β, through the approximate relation in Equation 4. In this equation Φ^{-1} represents the inverse of the standard normal distribution and p_f is the probability of failure

$$\beta = -\Phi^{-1}(p_f) \tag{4}$$

In the implementation of composite strengthening, failure through composite rupture is rarely witnessed, thus it was deemed unrealistic to compute the reliability of the beams against failure through composite rupture. Rather, the resistance of the beams was computed using the strain limitation provided by κ_m from ACI 440. This assumes that the ACI equation is a good predictor of the debonding strain, and implies that the reliability computed is the reliability against composite debonding (or very rarely concrete crushing), not against FRP rupture. As in the design stage, the strain limitation on the FRP resulted in concrete stress distributions that could not be accurately approximated using the common stress block factors in ACI 318. Thus the stress block factors for an assumed parabolic distribution were also used in the reliability evaluation.

RESULTS OF SAMPLE CALIBRATION

The resulting reliability indices are shown in Table 5. The results show that as the COV of the strength is increased there is very little difference in the reliability for load case 1. This is attributed to the high variation in the live load "drowning out" the effect of resistance variation. For load case 2 there is a clear decrease in reliability as the COV of strength is increased. This is one of the desired trends that the proposed method was selected to produce. For both load cases, as the resistance factor, ψ, is decreased the reliability of the beam is increased.

By selecting a target value for the reliability index, β_T, values of the resistance factor, ψ, needed to obtain this target can be found. For this example a target of 3.5 is used. Figures 1-2 show the deviation from the target reliability as a function of the resistance factor for load case 1. In Figure 1 the COV of strength is equal to 0.05, in Figure 2 it is 0.20. Here we see very little change in the required resistance factor as the strength COV is increased, implying that a single value of the resistance factor may be acceptable. Figures 3-6 show the deviation from target reliability as a function of the resistance factor for load case 2. In these figures it is clear that as the strength COV increases the resistance factor needed to reach the target reliability decreases. It is also clear that Material 3 shows significantly higher reliabilities than Materials 1 and 2. This indicates that the relative strength and stiffness of the composite can impact the reliability. The impact of higher strength variation and of higher strength and stiffness on reliability were not visible in load case 1 where the high live load variation dominated the reliability problem. However, load case 2 reveals these issues in reliability-based design. The values of ψ for load cases 1 and 2 are different from each other because case 2 is a specially selected example intended to highlight issues encountered in reliability based design.

A function of COV for determination of the composite resistance factor can be found for load case 2. For each COV the value of ψ necessary to provide a reliability of β_T is approximated from Figures 3-6. Figure 7 shows the equation fit to the relation between COV and ψ. In the range of COV from 0.05 to 0.10, ψ is constant at 0.8. For higher values of COV, ψ decreases linearly. The equation fitted to this line gives the value of ψ needed to reach the target reliability of 3.5 for COVs ranging from 0.05 to 0.20. A function such as this would be part of the design procedure and would allow designers to compute the resistance factor required to reach the target reliability specific to the COV that is anticipated in their design. This method could even allow designers to set their own target reliability index, by offering different functions for computing ψ for different values of the target reliability. Functions such as this would also allow designers to easily determine the effect on design of changing the COV through selection of higher quality materials or changes in quality control standards.

For all three materials at each level of COV the target value of reliability can be reached with use of the proper resistance factor, this shows that it is possible to use the mean value as the design value for composite material properties and still achieve safe designs as long as the proper resistance factor is used. For Materials 1 and 2, the relation between COV and reliability is very similar. However, Material 3 demonstrates different behavior. This change in behavior may be directly caused by the enhanced strength and stiffness of Material 3, or it may be a result of changes in the accuracy of the bond model at this higher stiffness. The cause for this change must be assessed. In order to make the proposed code widely applicable, it will be necessary to calibrate over a large range of possible properties. In order to select appropriate factors, it may be necessary to define ranges of properties where the behavior is similar and calibrate unique values of ψ for each of those ranges.

COMPARISON TO EXISTING DESIGN GUIDELINES

In order to highlight the advantages of the proposed procedure designs were also created with two existing approaches, ACI 440[1] and TR 55[2]. The ACI 440 design procedure requires that the characteristic value of the composite strength and ultimate strain be calculated as the mean less three standard deviations. This characteristic value is then multiplied by an environmental reduction factor, C_E, which in this case was taken to be 0.95 for a beam with interior exposure strengthened with CFRP. The ultimate strain is limited through κ_m, to prevent debonding. The design format includes an overall resistance factor of 0.9 and a composite specific reduction factor of 0.85. TR 55 defines the characteristic value as the mean less two standard deviations. Two safety factors are applied to the composite strength, one for the material, γ_{mF}, and one for the type of manufacture, γ_{mm}. In this example γ_{mF} was equal to 1.4 for CFRP and γ_{mm} was also set at 1.4 for wet layup manufacture. An additional factor, γ_{mE}, is applied to the modulus to account for long-term effects. Design using TR55 also used material partial safety factors of 1.15 for steel and 1.5 for concrete. For both the ACI 440 and TR55 designs FRP rupture or debonding controlled the design, and the concrete stress at that point was modeled using the stress block factors for a parabolic stress distribution. The resulting designs are summarized with the required composite width in Table 6. In many cases, particularly for the high COVs, it was not possible to create a design using these guidelines.

The calculated values of the reliability index for these designs are shown in Table 7. It is clear that there is significant variation in the reliability index. In particular, as the COV increases, so does the reliability (with the exception of TR55 designs at a COV of 0.20). This is the same result found by Atadero and Karbhari in [9]. The ACI 440 designs for strength COV equal to 0.05 are close to the target reliability of 3.5. However, it is important to note that while the design procedure took into account long-term effects, the reliability analysis did not consider time dependent degradation. Had degradation been considered all of the reliability indices in Table 7 would be lower, and many of these designs may have fallen below the target reliability.

It is important that a design code produces safe designs for all composite systems that may be designed using it. With the ACI 440 and TR55 design approaches, some of the designs are likely to be safe, but with such variation in reliability from case to case, it is impossible to assert that all designs will be safe. In contrast, reliability based design is specifically intended to explicitly consider material variability and its impact on the reliability. The sample calibration did not consider time-dependent behavior, however, this feature could be easily added and since the same principles would be used it would still be possible to provide relatively uniform reliability.

FURTHER WORK

The example presented here falls far short of a true calibration. In a true calibration a large collection of design cases would be represented, including beams of different sizes, various live to dead ratios and various degrees of strengthening. The intent of this example was simply to show the potential for COV specific resistance factors and the use of the mean as the design value. The example cases presented clearly demonstrate the need to consider many different design scenarios when calibrating a reliability-based code. For example, the differences witnessed between load cases 1 and 2 could also occur when the live to dead load ratio is very low.

In addition to the extended range of design cases that would be required for a full calibration, there are many other areas that require further development for creation of a complete code.

1. The design checking equation used in this example has two resistance factors, ϕ and ψ. For simplicity, only ψ was varied, however for a full code calibration ϕ could also be adjusted. The flexibility of two resistance factors may assist in making the code apply to a larger range of design cases.
2. Since the value of ψ depends on the COV it will be important to give designers a basis for estimating the COV the material is likely to exhibit in the field. This will require analysis of many sets of field data.
3. The limit state equations require further development. The present example looked only at flexural strengthening, and used a fairly simple equation to deal with the possibility of composite debonding. The reliability results found here suggest that stiffer composites may need to be treated slightly different from less stiff composites. Stiffer composites are more prone to debonding; however, if the bond model used for design is perfectly accurate, changes in stiffness should not affect the reliability of designs to the extent witnessed here. ACI 440 describes its bonding model as preliminary and likely to change with further research, thus it would be wise to settle on a more accurate model of debonding before dividing composite properties into different ranges of behavior.
4. One of the most important areas for further development is characterization of composite properties and the variation in those properties. The assumptions made here are based on the data available, but remain only assumptions. Full characterization of composite properties would include consideration of inherent variation; differences between test specimens and field specimens; and time-dependent degradation.

CONCLUSION

In this paper, an approach for creating a reliability based design code considering a variety of composite materials with different mean property values and different degrees of property variation is presented. This approach is based on the ideas of using the mean property values as the design values and having a composite specific resistance factor that is a function of the COV of the composite. A simple calibration example has shown the effectiveness of this approach in achieving uniform levels of reliability and how this code could be developed. This example shows, in at least a preliminary manner, the feasibility of the reliability based design approach presented. It also suggests that it is truly possible to create a reliability based design code for composite strengthening that applies to a wide range of composite materials.

Designs created using existing composite guidelines are also presented. It is shown that the existing guidelines have large degrees of variation in the reliability of their designs. This variation makes it impossible to assert the safety level of designs created with these guidelines without conducting complete reliability evaluations. As design moves toward risk based considerations it is important that the reliability can be accurately quantified.

Continuation of the code development process requires large amounts of further information. Research priorities are more information about composites including properties at manufacture and after exposure to environmental stresses and the development of more accurate models for various strengthening limit states. Efforts in this direction will be well worth the work, as a well-developed design code will make composite strengthening technology accessible to many engineers.

ACKNOWLEDGMENTS

The authors would like to acknowledge discussions with Dr. Charles Sikorsky and Susan Hida of Caltrans.

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Table 1 -- Load Effects on the Sample Beam

Load Effect	Existing Load	Anticipated Load
Unfactored Dead Load Effect	96.2 kN-m (72 k-ft)	96.2 kN-m (72 k-ft)
Unfactored Live Load Effect	114.9 kN-m (86 k-ft)	173.6 kN-m (130 k-ft)
Total Factored Moment	331.3 kN-m (248 k-ft)	428.8 kN-m (321 k-ft)

Table 2 -- Properties of Model Composites

Material	Ultimate Strength MPa (ksi)	Modulus GPa (ksi)	Thickness mm (in)
1	620.5 (90)	51.71 (7500)	2.03 (0.08)
2	827.4 (120)	68.95 (10000)	2.03 (0.08)
3	1034.2 (150)	86.18 (12500)	2.03 (0.08)

Table 3 -- Required Composite Width as a Function of ψ in mm (in)

ψ	Material 1	Material 2	Material 3
0.95	146 (5.75)	114 (4.5)	114 (4.5)
0.90	159 (6.25)	121 (4.75)	121 (4.75)
0.85	171 (6.75)	133 (5.0)	127 (5.0)
0.80	184 (7.25)	140 (5.5)	140 (5.5)
0.75	203 (8.0)	152 (5.75)	146 (5.75)
0.70	222 (8.75)	165 (6.25)	159 (6.25)
0.65	222 (9.75)	184 (6.75)	171 (6.75)
0.60	248 (11.25)	210 (7.5)	191 (7.5)
0.55	NA	241 (8.25)	210 (8.25)
0.50	NA	286 (9.5)	241 (9.5)

Table 4 -- Parameters of Statistical Distributions for FRP Properties

	Strength MPa (ksi)			Modulus GPa (ksi)		Thickness mm (in)	
	COV	α	β	λ	ζ	λ	ζ
MAT 1	0.05	24.950	634.23 (91.99)	3.9456 (8.9155)	0.1196	0.70658 (-2.5282)	0.0699
	0.1	12.153	647.23 (93.87)				
	0.15	7.907	659.29 (95.62)				
	0.2	5.797	670.15 (97.20)				
MAT 2	0.05	24.950	845.64 (122.65)	4.2333 (9.2032)	0.1196	0.70658 (-2.5282)	0.0699
	0.1	12.153	862.98 (125.16)				
	0.15	7.907	879.05 (127.50)				
	0.2	5.797	893.54 (129.60)				
MAT 3	0.05	24.950	1057.05 (153.31)	4.4565 (9.4263)	0.1196	0.70658 (-2.5282)	0.0699
	0.1	12.153	1078.72 (156.46)				
	0.15	7.907	1098.82 (159.37)				
	0.2	5.797	1116.93 (162.00)				

Table 5 -- Reliability Indices for Different Values of ψ and COV

		COV	ψ									
			0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
Load Case 1	Material 1	0.05	3.13	3.17	3.24	3.27	3.35	3.42	3.49	3.61	--	--
		0.10	3.12	3.17	3.23	3.27	3.34	3.41	3.49	3.58	--	--
		0.15	3.11	3.16	3.21	3.26	3.32	3.40	3.46	3.57	--	--
		0.20	3.09	3.14	3.19	3.22	3.29	3.36	3.43	3.54	--	--
	Material 2	0.05	3.12	3.16	3.23	3.26	3.33	3.38	3.46	3.58	3.69	3.84
		0.10	3.12	3.15	3.22	3.26	3.31	3.39	3.46	3.54	3.65	3.82
		0.15	3.10	3.14	3.19	3.22	3.29	3.36	3.44	3.54	3.64	3.80
		0.20	3.09	3.11	3.18	3.21	3.27	3.33	3.41	3.50	3.64	3.73
	Material 3	0.05	3.18	3.21	3.25	3.33	3.36	3.43	3.50	3.59	3.65	3.81
		0.10	3.17	3.21	3.25	3.31	3.35	3.43	3.51	3.58	3.65	3.79
		0.15	3.16	3.21	3.23	3.31	3.35	3.42	3.49	3.55	3.67	3.76
		0.20	3.15	3.18	3.23	3.29	3.33	3.40	3.44	3.56	3.64	3.75
Load Case 2	Material 1	0.05	3.19	3.31	3.41	3.50	3.62	3.72	3.83	4.00	--	--
		0.10	3.15	3.27	3.36	3.48	3.59	3.70	3.81	3.93	--	--
		0.15	3.09	3.20	3.30	3.39	3.52	3.66	3.76	3.92	--	--
		0.20	2.97	3.08	3.18	3.25	3.38	3.48	3.61	3.72	--	--
	Material 2	0.05	3.18	3.26	3.42	3.49	3.60	3.67	3.82	3.93	4.08	4.18
		0.10	3.15	3.24	3.37	3.46	3.59	3.67	3.81	3.93	4.02	4.16
		0.15	3.09	3.16	3.32	3.40	3.50	3.63	3.76	3.87	4.04	4.17
		0.20	3.01	3.07	3.20	3.26	3.38	3.47	3.58	3.73	3.81	3.94
	Material 3	0.05	3.32	3.41	3.49	3.66	3.77	3.85	3.98	4.05	4.14	4.19
		0.10	3.31	3.41	3.50	3.64	3.72	3.87	3.91	4.00	4.07	4.18
		0.15	3.29	3.38	3.46	3.59	3.67	3.78	3.85	3.99	4.09	4.23
		0.20	3.20	3.27	3.34	3.47	3.53	3.64	3.73	3.85	3.96	3.96

Table 6 -- Strengthening Designs Created with ACI 440 and TR55 mm (in)

	ACI 440			TR 55		
COV	Material 1	Material 2	Material 3	Material 1	Material 2	Material 3
0.05	203 (8.0)	165 (6.5)	159 (6.25)	248 (9.75)	203 (8.0)	165 (6.5)
0.1	248 (9.75)	203 (8.0)	197 (7.75)	--	235 (9.25)	191 (7.5)
0.15	--	260 (10.25)	260 (10.25)	--	273 (10.75)	216 (8.5)
0.2	--	--	--	--	305 (12.0)	260 (10.25)

Table 7 -- Reliability Indices for ACI 440 and TR55 Designs

	ACI 440			TR 55		
COV	Material 1	Material 2	Material 3	Material 1	Material 2	Material 3
0.05	3.63	3.69	3.83	3.84	3.89	3.91
0.1	3.80	3.92	4.07	--	4.04	4.02
0.15	--	4.04	4.18	--	4.13	4.17
0.2	--	--	--	--	3.97	4.03

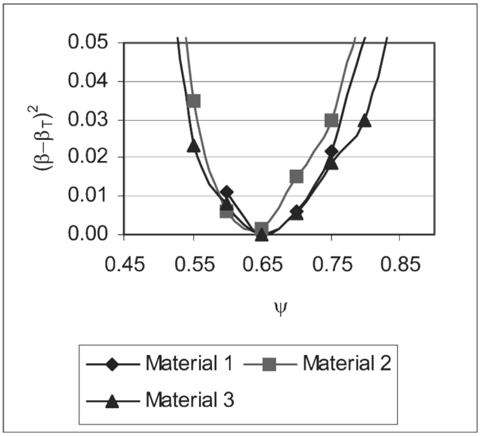

Figure 1 — Deviation from β_T as a Function of ψ for Load Case 1, COV=5.0%

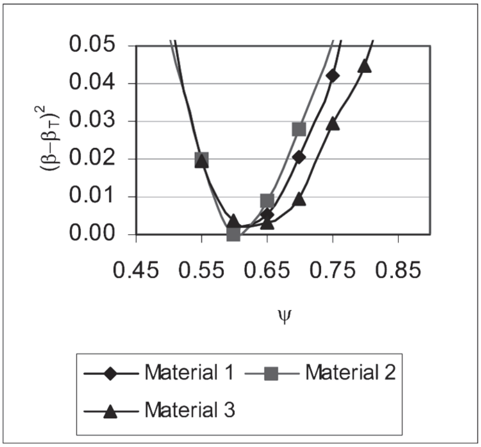

Figure 2 — Deviation from β_T as a Function of ψ for Load Case 1, COV=20.0%

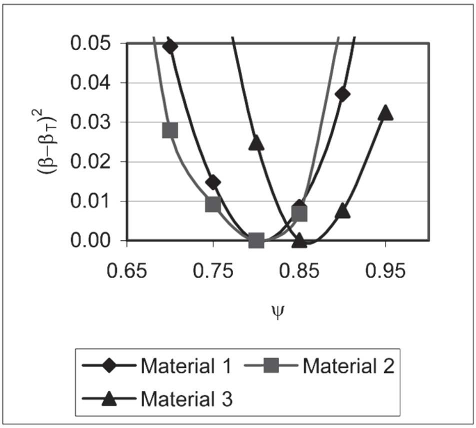

Figure 3 — Deviation from β_T as a Function of ψ for Load Case 2 COV=5.0%

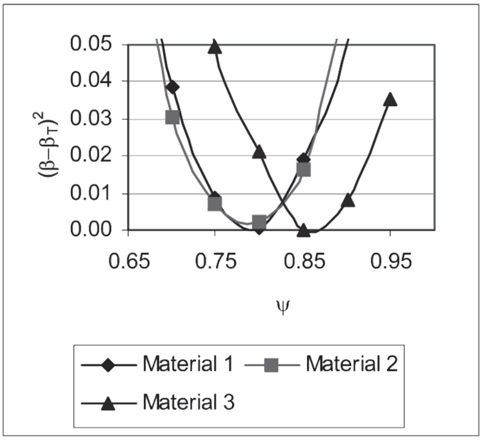

Figure 4 — Deviation from β_T as a Function of ψ for Load Case 2 COV=10.0%

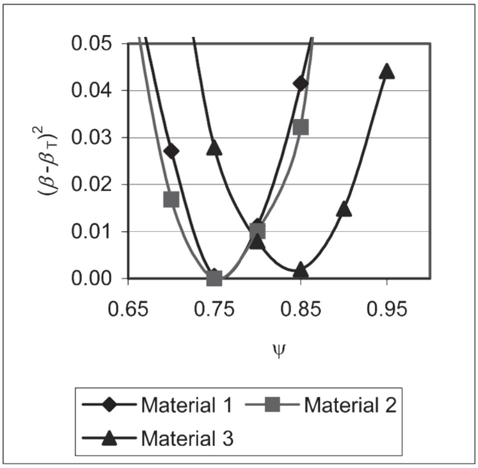

Figure 5 — Deviation from β_T as a Function of ψ for Load Case 2 COV=15.0%

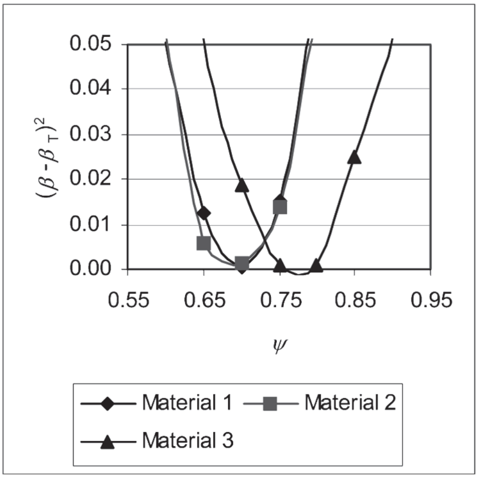

Figure 6 — Deviation from β_T as a Function of ψ for Load Case 2 COV=20.0%

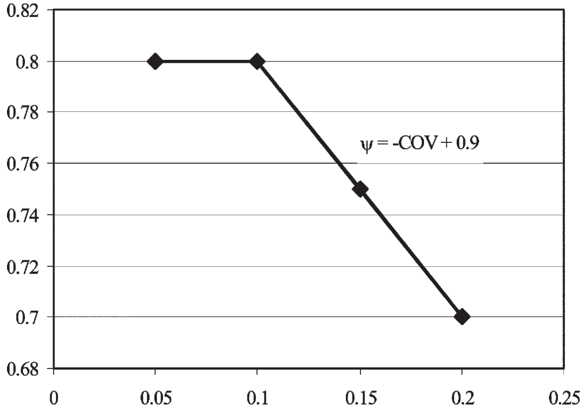

Figure 7 — ψ as a Function of the COV of Ultimate Strength for Materials 1 and 2, Load Case 2