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Influence of Bond Behavior on the Cross Sectional Forces in Flexural RC Members Strengthened with CFRP

by G. Zehetmaier and K. Zilch

Synopsis: In strengthened RC members the distribution of cross sectional tensile forces is affected by the significant differences in bond behavior of the reinforcement layers. As the tensile forces in the externally bonded reinforcement is the essential input value for bond verification for example at the end anchorage, the detailed knowledge of the distribution of forces in cracked sections is of fundamental importance. In this paper the common models to describe the interactions in tensile members are summarized and an advanced numerical model based on nonlinear bond stress-slip relationships for strengthened flexural members is presented. On the basis of experimental results combined with parametric studies, the effects of various parameters – for example the axial stiffness of CFRP, the diameter of internal rebars or the concrete compressive strength – on the interactions between the different reinforcement layers are examined. For practical design bond coefficients for a simplified calculation of cross sectional tensile forces are proposed.

Keywords: bond behavior; bond coefficients; cross-sectional forces; externally bonded reinforcement; flexural members; numerical model; tension stiffening

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INTRODUCTION

For flexural members the calculation of sectional forces assuming an even distribution of strains according to the *Bernoulli*-hypothesis is common practice. In general the *Bernoulli*-hypothesis leads to satisfying results if the bond behavior of the different reinforcement layers is equivalent. Tests on RC members strengthened with externally bonded reinforcement (EBR) show that there are significant discrepancies between measured tensile forces in the EBR and calculated values according to the *Bernoulli*-hypothesis (Ueda et al. 2002; Zehetmaier and Zilch 2003). These discrepancies are caused by the considerably different bond behavior of internal and externally bonded reinforcement. The resulting distribution of forces depends on the interaction of the different reinforcement layers. Like in RC members strengthened with EBR, in prestressed concrete members two reinforcement layers with different bond behavior are combined. According to design regulations, e.g. CEB FIP Model Code 1990 (CEB 1993) or EN 1992-1-1 (CEN 2004), the effects of the different bond characteristics have to be taken into consideration in serviceability and fatigue verification by the use of simplified bond coefficients.

RESEARCH SIGNIFICANCE

The tensile forces of the reinforcement layers are the essential input values e.g. for bond verification at the end anchorage or in the regions subjected to shear forces as well as for serviceability verification e.g. the calculation of crack widths. Therefore a detailed knowledge of the distribution of cross sectional tensile forces is of fundamental importance. In strengthened flexural members the interactions of internal and externally bonded reinforcement governs this distribution. In this paper the common models to describe interactions are summarized, an advanced model for flexural members strengthened with EBR is presented and proposals for bond coefficients with regard to a simplified calculation of cross sectional tensile forces are made.

INFLUENCE OF BOND ON CROSS SECTIONAL FORCES

Bond Behavior

As the interactions of different reinforcement layers are predominantly governed by the specific bond behavior, in preliminary studies bond models for externally bonded CFRP as well as embedded steel rebars have been established.

Externally bonded reinforcement – To represent the bond behavior of CFRP EBR a bilinear bond stress-slip relationship acc. to Fig. 1 has been derived from bond tests with double lap specimens. The basic parameters – the elastic energy G_e corresponding with the linear elastic behavior of bond, the interfacial fracture energy G_f and the bond shear strength τ_{f1} – have been determined by regression analysis of measured ε_f-s_f-relations using an approach presented in (Niedermeier and Zilch 2001) which in general is equivalent to the method published in (Dai et al 2005).

Internal rebars – To describe the bond behavior of embedded ribbed rebars numerous models are available. Based on the results of own bond tests the basic formulation acc. to (Eligehausen et al. 1983) was modified to represent splitting failure in case of small concrete cover. For smooth bars a rigid-plastic bond stress-slip relationship is assumed (Fig. 1). Effects influencing the bond behavior, e.g. the position during casting, are taken into account by means of reduction factors.

Influence of Bond on Tensile Members / Ties

The basic principles of the interaction of reinforcement layers with different bond characteristics may be illustrated on the basis of tensile members Fig. 2 a). The distribution of strains (ε_{fr}, ε_{sr}) or tensile forces (F_{fr}, F_{sr}) in cracked sections is strongly dependent on the bond behavior of the reinforcement layers and the specific ratio of axial stiffness to bond-effective circumference described with the shape coefficients $c_{s,f}$ and $c_{s,s}$ (Eq. 1). In addition to the mentioned parameters the stage of cracking and the crack spacing have significant impact on the distribution of cross sectional forces. A mechanical description of the interactions between different reinforcement layers can be established by applying equilibrium and compatibility conditions to a differential element. The resulting system of coupled differential equations represents a second order boundary value problem. Neglecting the influence of concrete deformations on the reinforcement strains and relative displacements, two independent differential equations Eqs. (2) and (3) are derived.

$$c_{s,f} = \frac{1}{E_f t_f}; \qquad c_{s,s} = \frac{4}{E_s d_s} \tag{1}$$

$$s_f'' - \tau_f(s_f) \cdot c_{s,f} = 0 \tag{2}$$

$$s_s'' - \tau_s(s_s) \cdot c_{s,s} = 0 \tag{3}$$

To solve the boundary value problem, bond stress-slip relationships $\tau_f(s_f)$, $\tau_s(s_s)$ and boundary conditions regarding strains or relative displacements (slip) are necessary.

The mechanical or numerical modeling of tensile members – commonly referred to as "tension stiffening models" – makes use of the symmetries for strain and slip distribution in the center between two cracks (Fig. 2 a) and the basic compatibility condition for the slip in cracked sections acc. to Eq. (4),

$$s_{sr} = s_{fr} = \frac{w_{cr}}{2} \tag{4}$$

where s_{sr} and s_{fr} are the relative displacements in the cracked section. As the crack width w_{cr} is supposed to be constant throughout the cross section, the usually assumed uniform crack spacing leads to a crack width that equals twice the relative displacement (Eq. 4). At present various models for strengthened tensile members exist. In (Holzenkämpfer 1994, Rostásy et al. 1996) an analytical model based on simplified rigid-plastic bond stress-slip relationships was presented. Numerical models taking into account nonlinear bond stress-slip relationships were published e.g. in (Sato et al 2002; Ulaga 2003; Pecce and Ceroni 2004).

Influence of Bond on Flexural Members

Tie models in combination with an effective area in tension may be used to describe the behavior of flexural members, but the so-called "tension chord models" are only valid for specific boundary conditions. Due to the assumption of symmetry between two cracks in conjunction with the compatibility of relative displacements at the cracks according to Eq. (4), the tension chord models may be appropriate to describe the interactions in regions with constant bending moment. The assumptions do not apply to regions subjected to shear forces (Fig. 2 b). In general, tension chord models assume equal inner lever arms for the different reinforcement layers. For the calculation of crack width or crack spacing, where only the total tensile force is of importance, the difference is negligible. As far as the distribution of tensile forces is concerned, tension chord models result in too small EBR forces, as usually the inner lever arm of EBR exceeds that of internal rebars. It is obvious that the differences in inner lever arms decrease with increasing member depth. But especially in case of slender slabs – at least in Germany the majority of strengthening applications - the effects of different distances to the neutral axis should be taken into account.

FLEXURAL MEMBERS – A NUMERICAL MODEL

To investigate the effects of bond behavior on cross sectional tensile forces in particular, a numerical model based on a modular program system has been developed. The model follows a discrete crack approach and allows the iterative calculation of strains (ε_f, ε_s) and relative displacements (s_f, s_s) along the entire length of the reinforcing elements on the basis of realistic nonlinear constitutive laws and bond models. Flexural members are represented by a sequence of macro-elements which idealize discrete segments e.g. between two adjacent cracks (Fig. 3).

Model Components

Element level (Fig. 3 b) – The behavior of strengthened flexural members at element level is characterized by material laws of concrete and reinforcement as well as bond models for internal and externally bonded reinforcement. The concrete compression zone ($\rightarrow F_c^{i,j}$) is described using a nonlinear material law acc. to (CEB 1993) implemented in a layer model. For reinforcing bars and CFRP laminates elastic-plastic and linear-elastic constitutive laws are used respectively. The bond behavior of externally bonded CFRP reinforcement and internal rebars is described with the already introduced nonlinear bond models. Local effects, e.g. local bond degradation in the immediate vicinity of initial flexural cracks due to the formation of diagonal cracks caused by the EBR or the cone shaped cracks surrounding the embedded bar have an impact on the interactions. To account for local effects, correction factors for the compatibility conditions (see Eq. 9) depending on d_s, t_f and f_{cm} were derived from 56 tests on strengthened RC tensile members (Zehetmaier and Zilch 2003). This allows the use of global bond models in numerical calculations as well as in analytical solutions for bond coefficients. The bond models do not include hysteretic behavior. Effects resulting from the reversal of relative displacement between reinforcement and concrete due to the formation of a new crack or the incremental shift of the points of zero slip ($s_f = 0$ and $s_s = 0$ respectively, see Fig. 6) with increasing mean strain are not considered.

System level (Fig. 3 a) – In contrast to tensile members, for flexural members no boundary conditions at element level – expressed e.g. with a fixed relation between s_f and s_s – exist. Boundary conditions may only be formulated at system level for specific cross sections, e.g. at the system symmetry axis (Eq. 5) or at the ends of the reinforcing elements (Eq. 6) (see Fig. 3 for notation)[a].

$$s_{sr}^{n,2} = s_{sr}^{n+1,1} \qquad s_{fr}^{n,2} = s_{fr}^{n+1,1} \tag{5}$$

$$\begin{aligned} s_s(x_s = 0) &\neq 0 \qquad \varepsilon_s(x_s = 0) = 0 \\ s_f(x_f = 0) &\neq 0 \qquad \varepsilon_f(x_f = 0) = 0 \end{aligned} \tag{6}$$

The discrete macro-elements are linked with equilibrium and compatibility conditions. As the Bernoulli-hypothesis is not valid, there are three independent constraints referring to cracked sections (i.e. the elements boundaries):

- equilibrium of sectional forces and action effects acc. to Eq. (7)

$$\begin{aligned} \sum N^{i,j} &\Rightarrow F_{fr}^{i,j} + F_{sr}^{i,j} + F_c^{i,j} - N^{i,j} = 0 \\ \sum M^{i,j} &\Rightarrow F_{fr}^{i,j} z_{fr}^{i,j} + F_{sr}^{i,j} z_{sr}^{i,j} - M^{i,j} = 0 \end{aligned} \tag{7}$$

- compatibility of strains acc. to Eq. (8)

[a] The first superscript (i) denotes the element (element number $1 \leq i \leq n$), the second superscript denotes the element boundary ($j = 1,2$) (see Fig. 3)

$$\varepsilon_{fr}^{i,2} = \varepsilon_{fr}^{i+1,1} \qquad \varepsilon_{sr}^{i,2} = \varepsilon_{sr}^{i+1,1} \tag{8}$$

- compatibility of relative displacements acc. to Eq. (9)

$$\left(s_{sr}^{i,2} + s_{sr}^{i+1,1}\right) \cdot k_x^{i,i+1} = s_{fr}^{i,2} + s_{fr}^{i+1,1} \tag{9}$$

In Eq. (9) $k_x^{i,i+1}$ takes into account the influence of different distances to the neutral axis (i.e. the influence of curvature) and includes the correction factors to consider local bond effects. Eq. (9) may be interpreted as a compatibility condition of crack widths at the z-coordinates of the reinforcement layers (Fig. 3 c). The mentioned independent constraints are used to control the numerical calculation and to formulate convergence conditions respectively.

As mentioned above, the distribution of cross sectional forces depends on the crack pattern and particularly on the spacing between two adjacent cracks. To reduce complexity in the established numerical model a predefined crack pattern with constant crack spacing is assumed. The mean crack spacing equals 1.4 l_t where l_t is the transfer length resulting from the cracking moment; l_t is calculated based on the presented bond models. The factor 1.4 was obtained from the evaluation of stabilized crack patterns observed in tests with strengthened tensile and flexural members and takes into account the differences between the calculated initial crack spacing ($=s_{cr,max} = 2\ l_t$) and the mean crack spacing at stabilized cracking.

Comparison with experimental results

To identify effects of bond behavior on the distribution of cross sectional tensile forces an extensive experimental program covering tests on strengthened ties and flexural members was conducted at the Technical University Munich (Zehetmaier and Zilch 2003). The comparison of measured FRP strain in a predefined crack and calculated strain assuming even cross sections (*Bernoulli*-hypothesis) displayed in Fig. 4 a demonstrates the significant influences of bond behavior. Especially at low loading range (serviceability level) the measured strains exceed the values calculated on the basis of the *Bernoulli*-hypothesis by 80%. In contrast to the *Bernoulli*-hypothesis the F-ε_{fr}-relation in eight predefined cracks calculated with the presented numerical model is in good agreement with measured values (Fig. 4 a, b). Of course the distribution of tensile forces is fixed if the internal reinforcement yields (Fig. 4 a). Various tests on strengthened flexural members reported in literature, where reinforcement strains in discrete cracks were available, were recalculated. Although in absence of a predefined crack pattern successive cracking occurred, in general the calculated values based on the mean crack spacing are in good accordance with measured data.

Due to the general material laws and bond models, the complete loading range from serviceability to ultimate limit state may be followed. In general bond failure of EBR is initiated when the required increase in tensile force between two cracks exceeds the bond capacity. In Fig. 5 the distribution of strains of internal and externally bonded reinforcement along half of the system is displayed for the last convergent load increment. The bond failure of EBR will initiate in element 12 as there the bond capacity is reached. It should be noted that the location where bond failure starts according to the numerical model corresponds to observations in tests. In Fig. 6 the calculated distribution of strains and relative displacements for four different load steps in case of an element in the shear span is displayed. The lacking symmetries at element level as well as the shift of the points of zero slip are obvious.

BOND COEFFICIENTS

Background

Like in the design of PC members, the effects of different bond behavior on tensile forces in cracked sections may be taken into consideration by means of simplified bond coefficients. As the coefficients should be independent from specific reinforcement ratios or distances to the neutral axis a formulation according to Eq. (10) was chosen. Obviously δ_f equals 1 if the *Bernoulli*-hypothesis is accurate.

$$\delta_f = \frac{\varepsilon_{fr}}{\varepsilon_{sr}} \cdot \frac{d-x}{d_f - x} \tag{10}$$

Due to the formulation the bond coefficients will be referred to as "strain ratio". From equilibrium of internal forces a relationship between ε_{fr} and ε_f^{II} is derived (Eq. 11), where ε_f^{II} is the EBR strain in the cracked section calculated acc. to the *Bernoulli*-hypothesis. With predetermined bond coefficients δ_f the realistic strain ε_{fr} can be determined with Eq. (11). For deep beams or ties where *$(d_f$-x)/(d-x)* approaches 1, the resulting expression – Eq. (11-right) – corresponds to the equation given in (CEB 1993) for the design of PC members.

$$\varepsilon_{fr} = \frac{\left(1 + \frac{E_f A_f}{E_s A_s} \cdot \frac{z_f}{z_s} \cdot \frac{d_f - x}{d - x}\right) \cdot \delta_f}{1 + \frac{E_f A_f}{E_s A_s} \cdot \frac{z_f}{z_s} \cdot \frac{d_f - x}{d - x} \cdot \delta_f} \cdot \varepsilon_f^{II} \quad \Rightarrow \quad \varepsilon_{fr} = \frac{\left(1 + \frac{E_f A_f}{E_s A_s}\right) \cdot \delta_f}{1 + \frac{E_f A_f}{E_s A_s} \cdot \delta_f} \cdot \varepsilon_f^{II} \tag{11}$$

Parameters governing tensile forces

The numerical model can be used to estimate the influence of various parameters on the distribution of tensile forces in cracked sections. The results of parametric studies displayed in Fig. 7 are based on the simply supported slab acc. to Fig. 6. The resulting strain ratio δ_f at midspan is plotted versus the corresponding strain ε_f^{II}. The following general tendencies may be derived (note: $\delta_f > 1$ means that the EBR tensile strain exceeds the value calculated acc. to the *Bernoulli*-hypthesis):

- Due to the softening behavior of EBR bond the strain ratio decreases with increasing load. The pronounced maximum refers to single cracking, i.e. the idealized transfer lengths of internal and external reinforcement are smaller than half the crack spacing;

- With increasing bar diameter or decreasing axial stiffness of FRP EBR the strain ratio δ_f increases (Fig. 7 a, b);
- The impact of the concrete compressive strength on the strain ratio is limited to service loads (Fig. 7 c) although there is a dominant correlation between compressive or tensile strength and bond capacity;
- The crack spacing is governing the initiation of debonding as well as the redistributions of forces to the internal reinforcement, but it is only of little importance regarding the maximum strain ratio (Fig. 7 d).

The results of parametric studies in Fig. 7 demonstrate that, in contrast to the design of PC members, constant bond coefficients will only be a roughly approximate representation of the real interaction behavior. The complex interaction is mainly due to the softening behavior of EBR bond.

Concept for Bond Coefficients

As a consequence of the complex interaction behavior, bond coefficients may only be determined for specific boundary conditions e.g. the single cracking stage. Due to the used global bond models analytical expressions can be established depending on the parameters of the bond stress-slip relationships and geometrical variables. In Fig. 8 two different types of bond coefficients for the combination of externally bonded CFRP strips and ribbed reinforcing bars are displayed. The coefficient $\delta_{f,max}$ (Fig. 8 a) corresponds to single cracking and should be used e.g. for fatigue verification of bond at serviceability level. For bond verification in ULS the coefficients $\delta_{f,e}$ (Fig. 8 b) are appropriate.

CONCLUDING REMARKS

In this paper the fundamental characteristics of a numerical model are summarized which allows to examine RC members strengthened in flexure with externally bonded CFRP. Comparisons with test results confirm the reliability of calculations. Based on numerical results the dominant parameters influencing the interaction of the different reinforcement layers are identified and bond coefficients to simplify the calculation of cross sectional tensile forces are established. As the distribution of strains and in particular the relative displacements at the cracked sections are connected to the element curvature, the numerical model also allows to calculate the flexural deformations of strengthened members. Currently improvements to take into account crack formation and crack spacing are implemented in the program system.

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NOTATION

The following symbols are used:

A_f	cross section (EBR)	h	depth
A_s	cross section (rebars)	k_x	curvature coefficient
E_c	Young's modulus (concrete)	s_{cr}	crack spacing
E_f	Young's modulus (EBR)	s_f	slip (EBR)
E_s	Young's modulus (rebars)	s_s	slip (rebars)
F_c	concrete compressive force	t_f	thickness of EBR
F_{fr}	EBR force (cracked section)	w_{cr}	crack width
F_{sr}	Rebar force (cracked section)	x	depth of compression zone
G_e	elastic bond energy (EBR)	z_f	inner lever arm (EBR)
G_f	bond fracture energy (EBR)	z_s	inner lever arm (rebars)
M	bending moment	δ_f	bond coefficient ("strain ratio")
N	axial force	ε_f	strain (EBR)
$c_{s,f}$	shape coefficient (EBR)	ε_{fr}	strain in the cracked section (EBR)
$c_{s,s}$	shape coefficient (rebars)	ε_f^{II}	strain acc. to Bernoulli-hypothesis
d	effective depth (rebars)	ε_s	strain (rebars)
d_f	effective depth (EBR)	ε_{sr}	strain in the cracked section (rebars)
d_s	rebar diameter	τ_f	bond stress (EBR)
f_{cm}	concrete compressive strength	τ_{f1}	bond strength (EBR)
f_{ctm}	concrete tensile strength	τ_s	bond stress (rebars)

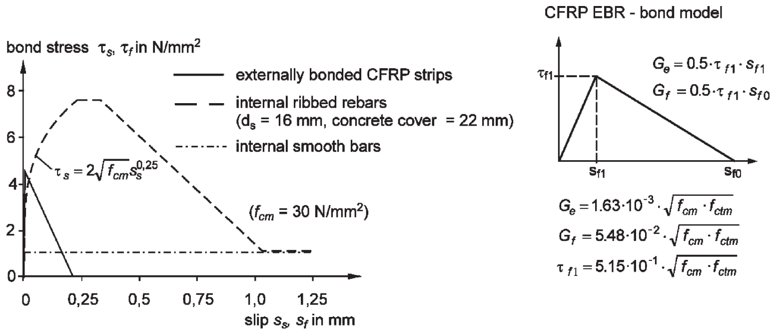

Figure 1 – Bond models for internal rebars and externally bonded reinforcement

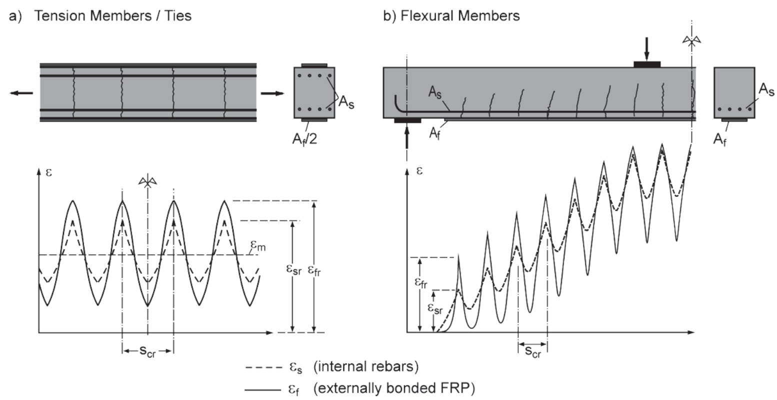

Figure 2 – Influence of bond on tensile strains in the different reinforcement layers

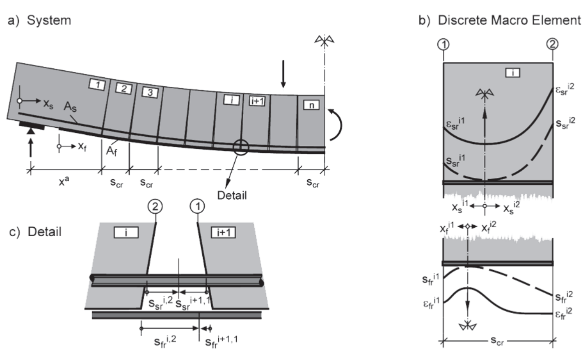

Figure 3 – Numerical model for strengthened flexural members

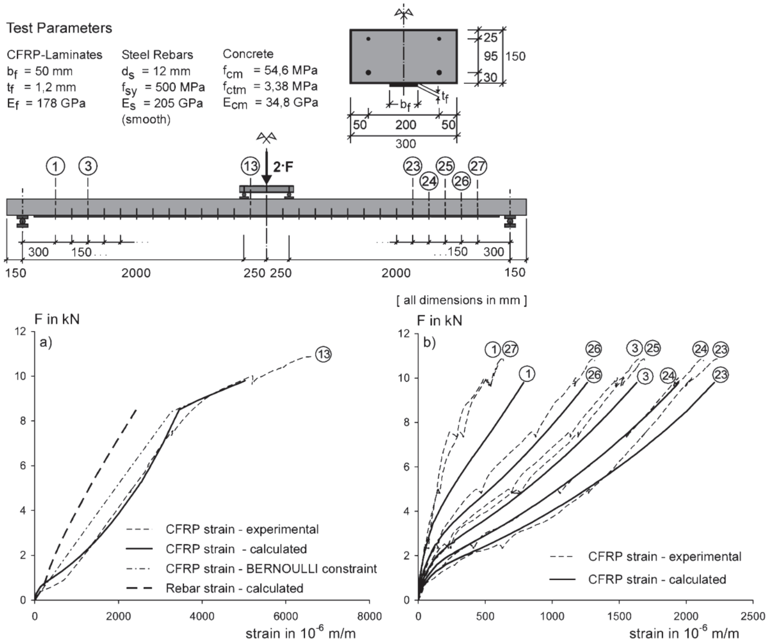

Figure 4 – Calculated reinforcement strains vs. measured strains in predefined cracks (slab No. B2-08-B2C)

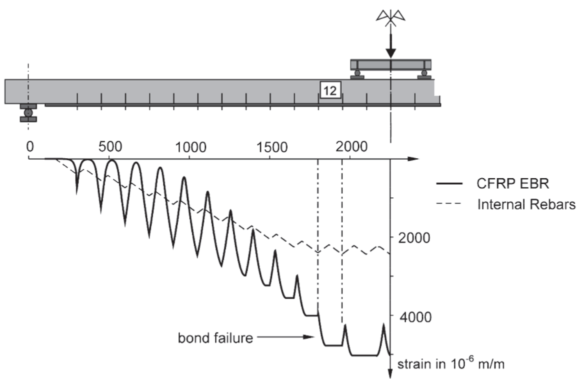

Figure 5 – Calculated reinforcement strains prior to bond failure (last convergent increment) (slab No. B2-08-B2C)

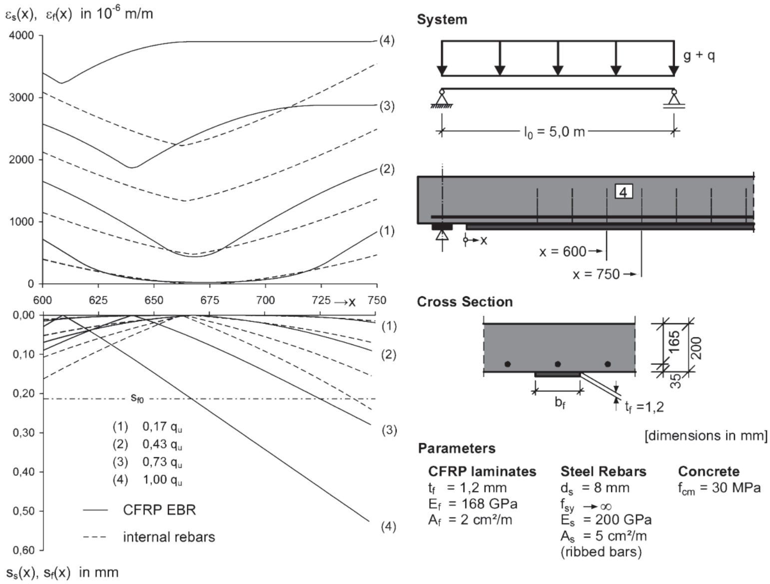

Figure 6 – Reinforcement strains and related slip in element No. 4 for 4 different load steps

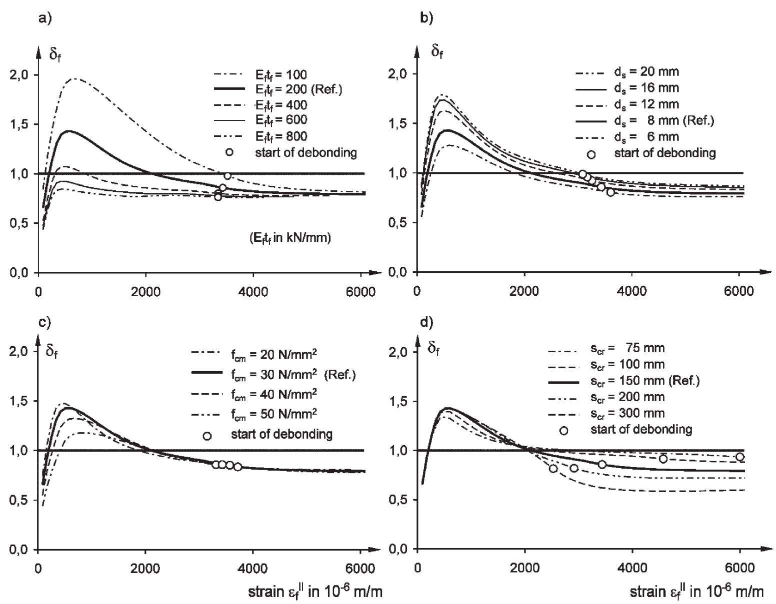

Figure 7 – Development of strain ratio d_f at midspan with increasing load, parameters affecting the distribution of cross sectional forces (Reference system acc. to Fig. 6)

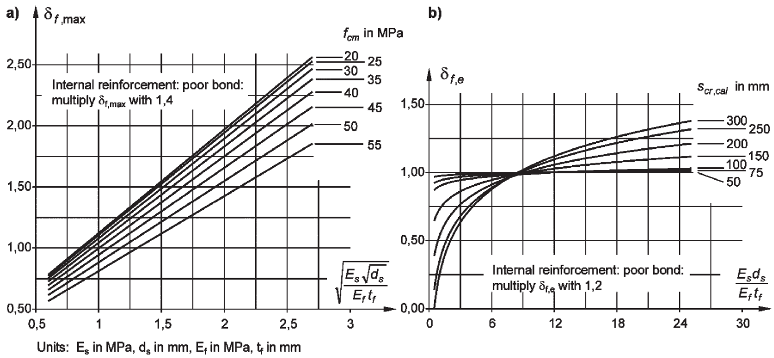

Figure 8 – Bond coefficients for CFRP-laminates combined with ribbed reinforcing bars