Design Guidelines for Flexural Strengthening of RC Beams with FRP Plates

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Abstract: Bonding composite plates to reinforced concrete beams is an effective technique of repair and retrofit. The ultimate capacity of the strengthened beam is controlled by either compression crushing of concrete, rupture of the plate, local failure of concrete at the plate end due to stress concentration, or debonding of the plate. These failure modes have been considered for developing design guidelines for strengthening reinforced concrete beams using fiber composite plates. The effect of multistep loading of the beam, before and after upgrading, has been included in these guidelines. Limit state design concept has been followed in this paper. Terms, definitions, and notations compatible to design guidelines for ordinary reinforced concrete beams have been used.

INTRODUCTION

Fiber reinforced plastic (FRP) plates or fabrics are becoming increasingly popular materials for strengthening of reinforced concrete (RC) beams and girders. This strengthening technique involves epoxy bonding FRP plates or fabrics to the tension face of the beam, increasing both the strength and stiffness of the beam, as shown in Fig. 1. It is noted that the concrete surface must be cleaned or sandblasted down to aggregate to assure a good bond between the FRP plate and concrete. Several advantages of these materials, such as formability, ease of fabrication and bonding, corrosion resistance, and light weight have attracted the attention of many engineers involved in retrofit design. Depending on the required physical and mechanical properties, and also economic considerations, composites can be made from a variety of resins and fibers (Malek and Saadatmanesh 1996) and tailored to provide the necessary strength and stiffness in the desired direction (Jones 1975). FRP generally behave linearly elastic up to failure, if loaded in the fiber direction.

In studying the behavior of RC beams strengthened with FRP plates, different types of failure modes have been reported (Ritchie et al. 1991; Saadatmanesh and Ehsani 1991). Ultimate strength of the beam is generally controlled by rupture of the plate or compression crushing of concrete (An et al. 1991). However, local failure in concrete beam at the plate end due to stress concentrations or debonding of the plate may also lead to premature failure of the strengthened beam. Shear and normal (peeling) stress concentrations at the cut-off point or around flexural cracks are the main reason for local failures. Since in many cases local failure occurs at the plate cut-off point, this paper addresses only the local failure at this point. Additional studies need to be conducted to examine the effect of stresses at the flexural crack tip on the failure of the beam. Closed form solutions have been developed to predict stress concentrations at the plate ends leading to these local failures (Malek 1997; Malek et al. 1998). Fig. 2 shows failure of the beam as a result of failure of the concrete layer between the FRP plate and the longitudinal steel reinforcement, initiated by the stress concentrations near the cracks and at the plate end.

This paper presents design guidelines for strengthening of simply supported RC beams with FRP plates. For continuous beams, a similar concept can be used to develop appropriate design guidelines. The effect of initial stresses in the concrete beam before bonding the plate has been considered in the design. The same terminology and notations as those of ordinary RC beams have been used throughout this paper to facilitate their application by practicing engineers. A practical design example has also been presented to highlight the different steps involved in application of these guidelines.

EFFECT OF INITIAL STRESSES ON ULTIMATE CAPACITY OF STRENGTHENED BEAM

The effect of the stresses that the RC beam undergoes before bonding the plate must be considered in calculation of the ultimate flexural capacity of the strengthened beam. To simplify the design process, it is assumed that the strain in the plate and the concrete at the interface are equal. However, it is noted that in reality the strain in the concrete at the interface is higher due to the initial stresses. To account for this discrepancy, the normal strain in the composite plate is reduced to include the effect of initial strains and to allow direct ap-

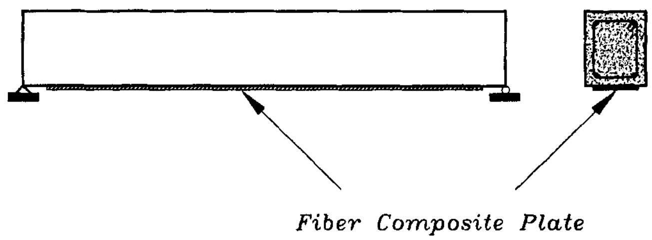

FIG. 1. Flexural Strengthening of RC Beams with Composite Plates

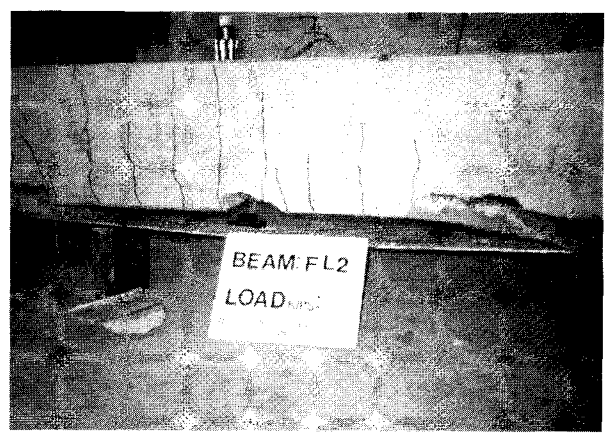

FIG. 2. Failure of Concrete Layer between FRP Plate and Tension Reinforcement

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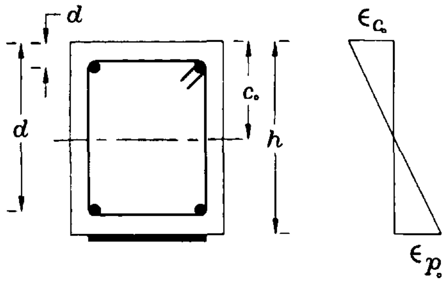

FIG. 3. **Strain Diagram of Concrete Beam before Strengthening**

plication of linear strain variation along the section of the strengthened beam. If the service moment (without any load factors) acting on the RC beam before upgrading is defined by M_o, then strain at the top of the concrete beam can be expressed by

$$\varepsilon_{c_o} = \frac{M_o c_o}{E_c I_{tro}} \tag{1}$$

where c_o = depth of neutral axis (Fig. 3); I_{tro} = moment of inertia of the transformed cracked section based on concrete; and E_c = modulus of elasticity of concrete. Considering linear variation of strains along the section of the beam, tensile strain at the bottom face of the concrete, ε_{p_o}, where the composite plate is to be bonded, is obtained from

$$\varepsilon_{p_o} = \varepsilon_{c_o}\left(\frac{h - c_o}{c_o}\right) \tag{2}$$

In calculating the stress in the composite plate, the above strain is subtracted from the total strain computed based on linear variation of strains in the cross section. In the previous expression, the effect of the thickness of the composite plate has been ignored to simplify the design equations. It is noted that the plate thickness is generally much smaller than the height of the beam, justifying this approximation. Therefore, the actual stress in the composite plate is calculated from

$$f_p = E_p(\varepsilon_p - \varepsilon_{p_o}) \tag{3}$$

where f_p = normal stress in the composite plate; E_p = modules of elasticity of the plate in the fiber direction; and ε_p = axial strain at the level of the plate based on linear strain variation in the strengthened beam. Furthermore, in all derivations that follow, it is assumed that full composite action exists between the plate and the beam, i.e., no slip at the interface.

ULTIMATE FLEXURAL CAPACITY OF STRENGTHENED BEAM

This method of strengthening is generally used for two purposes.

1. For upgrading the strength of beams beyond their existing levels to functional changes, increase in permit loads, etc. The strengthening applied to these cases will be referred to by "retrofit" in this paper.
2. For beams with insufficient reinforcement due to various reasons such as design errors, omission of rebars during construction, or loss of steel area caused by corrosion. The strengthening applied to these cases will be referred to by "Repair" in this paper.

Only two modes of failure are considered for calculating the ultimate capacity; namely, compression crushing of concrete and FRP plate rupture. Other modes of failure, i.e., local failure in the concrete layer between tension reinforcement and composite plate, and debonding of the plate around the flexural cracks, are checked and avoided by choosing appropriate dimensions for the composite plate.

Balanced Plate Ratio for Steel Yielding

This ratio gives the maximum cross-sectional area of the plate to assure yielding of the steel reinforcement at the time of concrete crushing. Under balanced conditions (Fig. 4), and based on linear strain variation, the balanced plate ratio where steel yields at the same time that concrete crushes is obtained by equating the resultant compressive and tensile forces in the cross section and solving for the balanced area of the plate. The balanced plate ratio is then obtained by dividing the area of the plate by the product "b.d" as given in the following equations

$$\rho_{p,b} = \frac{\acute{\rho} f_y + 0.85 f'_c \beta_1 \eta_1 - \rho f_y}{\left(\varepsilon_u \dfrac{h - \eta_1 d}{\eta_1 d} - \varepsilon_{p_o}\right) E_p} \quad \text{if compression steel has yielded} \tag{4}$$

$$\rho_{p,b} = \frac{\left(\dfrac{\eta_1 d - \acute{d}}{\eta_1 d}\right) E_s \acute{\rho} + 0.85 f'_c \beta_1 \eta_1 - \rho f_y}{\left(\varepsilon_u \dfrac{h - \eta_1 d}{\eta_1 d} - \varepsilon_{p_o}\right) E_p}$$

if compression steel has not yielded (5)

In the above equations

$$\eta_1 = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \tag{6}$$

where ε_u = ultimate strain in the concrete (generally = 0.003); ε_y = yield strain of the steel reinforcement; f_y = yield stress of steel reinforcement; E_s = modules of elasticity of steel; f'_c = compressive strength of concrete; and β_1 is the rectangular stress block parameter defined in the ACI code (Nilson 1997). Furthermore

$$\rho_p = \frac{A_p}{bd} \quad \rho = \frac{A_s}{bd} \quad \acute{\rho} = \frac{\acute{A}_s}{bd}$$

where A_p, A_s, and $\acute{A}_s$ are the cross-sectional areas of the composite plate, tension reinforcement, and compression reinforcement, respectively.

In the balanced condition, compression reinforcement will yield provided that the following condition is satisfied:

$$\acute{d} \le \left(\frac{\varepsilon_u - \varepsilon_y}{\varepsilon_u + \varepsilon_y}\right) d \tag{7}$$

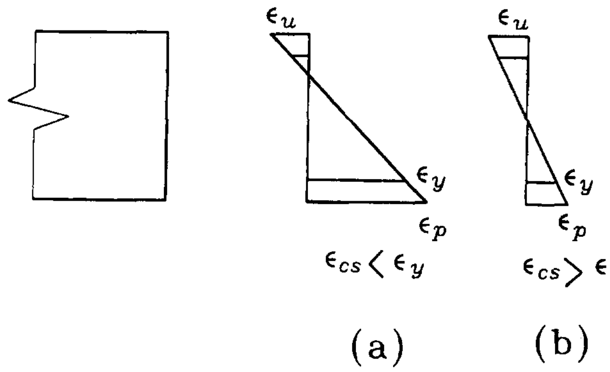

FIG. 4. **Strain Variation at Balanced Plate Ratio for Steel Yielding: (a) Compression Steel Has Not Yielded; (b) Compression Steel Has Yielded**

The determination of the maximum allowable plate ratio, $\rho_{p,\max}$, is beyond the scope of this study. However, in the absence of such investigations, it is recommended to use the American Concrete Institute (ACI) factor of 0.75 as shown in the following equation:

$$\rho_{p,\max} = 0.75\rho_{p,b} \tag{8}$$

Yielding of Compression Steel at Ultimate Case

Using linear strain diagram and also the corresponding stress diagram (Fig. 5), the minimum plate ratio for yielding of the compression steel is calculated. Two modes of failure will be considered only: (1) composite plate rupture; and (2) concrete crushing. One of the following equations is used to find the critical plate ratio, where compression steel yields at or before beam failure.

Composite Plate Rupture

$$\rho_{p,cy} = \frac{0.85\acute{f}_c\beta_1 \dfrac{c}{d} + \acute{\rho} f_y - \rho f_y}{f_{pr}} \tag{9}$$

where $c = (\varepsilon_y h + \varepsilon_r \acute{d})/(\varepsilon_r + \varepsilon_y)$; f_{pr} = composite plate stress at rupture; and $\varepsilon_r = f_{pr}/E_p$ is the ultimate strain in the composite plate at rupture. If the plate ratio exceeds the value of $\rho_{p,cy}$, the compression steel reinforcement will yield at the ultimate load level of the strengthened beam.

Concrete Crushing

$$\rho_{p,cy} = \frac{0.85\acute{f}_c\beta_1\eta_2 \dfrac{\acute{d}}{d} + (\acute{\rho} - \rho) f_y}{E_p(\varepsilon_p - \varepsilon_{p_o})} \tag{10}$$

where $\eta_2 = \varepsilon_u/(\varepsilon_u - \varepsilon_y)$; and $\varepsilon_p = \varepsilon_u(h - \eta_2\acute{d})/\eta_2\acute{d}$. The ultimate capacity reached as a result of local failure of concrete or debonding of the plate is independent of the stress level in the compressive steel; therefore, $\rho_{p,cy}$ is not defined for these modes of failure.

Nominal Moment Capacity of Strengthened Beam

The failure of the strengthened beam may result from crushing of concrete or rupture of the plate. The condition at which the maximum compressive stress in concrete and tensile stress in the composite plate reach their ultimate values at the same time is here referred as "balanced condition" (Fig. 6). Defining the required plate ratio for this mode of failure by $\rho_{p,bb}$, and using the linear strain diagram, the balanced plate ratio is calculated as

$$\rho_{p,bb} = \frac{0.85\acute{f}_c\beta_1 \dfrac{\eta_3 h}{d} + \acute{\rho}\acute{\varepsilon}_s E_s - \rho f_y}{f_{pr}} \tag{11}$$

where $\eta_3 = \varepsilon_u/(\varepsilon_u + \varepsilon_r + \varepsilon_{p_o})$. In the above equation $\acute{\varepsilon}_s$ is strain in the compression steel reinforcement and is calculated using the following equation:

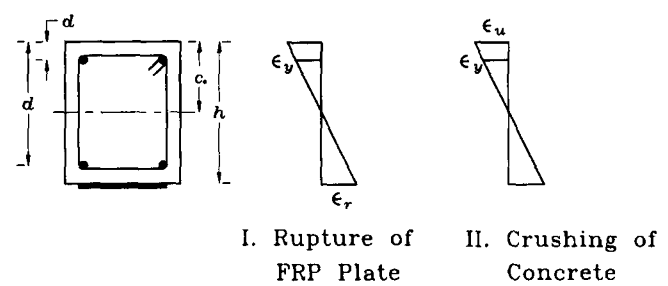

FIG. 5. Strain Diagram at Failure of Strengthened Beam

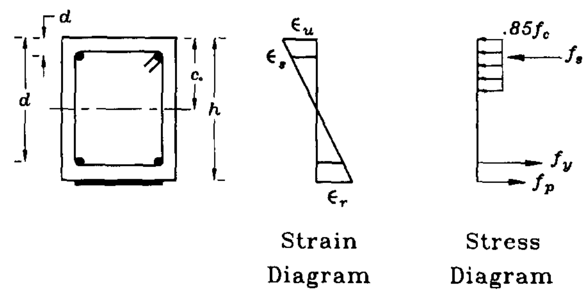

FIG. 6. Strain and Stress Diagrams of Strengthened Beam at Balanced Conditions

$$\acute{\varepsilon}_s = \left(1 - \frac{\acute{d}}{\eta_3 h}\right)\varepsilon_u \le \varepsilon_y \tag{12}$$

In cases where $\rho_p \le \rho_{p,bb}$, failure of the strengthened beam is caused by the rupture of the plate, otherwise it is caused by crushing of concrete in the compression zone.

Based on the above failure modes, the nominal flexural capacity of the strengthened beam, M_n, is calculated using one of the following equations:

Rupture of Plate

Compression steel yields at ultimate load ($\rho_p \ge \rho_{p,cy}$)

$$M_n = \acute{A}_s f_y\left(\frac{\beta_1 c}{2} - \acute{d}\right) + A_s f_y\left(d - \frac{\beta_1 c}{2}\right) + A_p f_{pr}\left(h - \frac{\beta_1 c}{2}\right) \tag{13}$$

where $c = (A_s f_y + A_p f_{pr} - \acute{A}_s f_y)/0.85\acute{f}_c b\beta_1$.

Compression steel does not yield at ultimate load ($\rho_p \le \rho_{p,cy}$)

$$M_n = \left(\frac{c - \acute{d}}{h - c}\right)(\varepsilon_r + \varepsilon_{p_o})\acute{A}_s E_s\left(\frac{\beta_1 c}{2} - \acute{d}\right) + A_s f_y\left(d - \frac{\beta_1 c}{2}\right) + A_p f_{pr}\left(h - \frac{\beta_1 c}{2}\right) \tag{14}$$

where the depth of the neutral axis c is calculated using the following quadratic equation:

$$\bar{A}c^2 + \bar{B}c + \bar{C} = 0 \tag{15}$$

where

$$\bar{A} = 0.85\acute{f}_c\beta_1 b$$

$$\bar{B} = -0.85\acute{f}_c\beta_1 hb - (\varepsilon_r + \varepsilon_{p_o})\acute{A}_s E_s - A_s f_y - A_p f_{pr}$$

$$\bar{C} = (\varepsilon_r + \varepsilon_{p_o})E_s\acute{A}_s\acute{d} + (A_s f_y + A_p f_{pr})h$$

Compression Crushing of Concrete

Compression steel yields at ultimate load ($\rho_p \ge \rho_{p,cy}$)

$$M_n = \acute{A}_s f_y\left(\frac{\beta_1 c}{2} - \acute{d}\right) + A_s f_y\left(d - \frac{\beta_1 c}{2}\right) + \left(\frac{h - c}{c}\varepsilon_u - \varepsilon_{p_o}\right) \cdot E_p A_p\left(h - \frac{\beta_1 c}{2}\right) \tag{16}$$

where c is found using (15) with the following parameters:

$$\bar{A} = 0.85\acute{f}_c\beta_1 b$$

$$\bar{B} = (\acute{A}_s - A_s)f_y + (\varepsilon_u + \varepsilon_{p_o})E_p A_p$$

$$\bar{C} = -\varepsilon_u h A_p E_p$$

Compression steel does not yield at ultimate load ($\rho_p \le \rho_{p,cy}$)

$$M_n = \left(\varepsilon_u \frac{c - \acute{d}}{c}\right) E_s \acute{A}_s \left(\frac{\beta_1 c}{2} - \acute{d}\right) + A_s f_y \left(d - \frac{\beta_1 c}{2}\right)$$

$$+ \left(\frac{h - c}{2} \varepsilon_u - \varepsilon_{p_o}\right) E_p A_p \left(h - \frac{\beta_1 c}{2}\right) \quad (17)$$

where c is calculated using (15) and the following parameters:

$$\bar{A} = 0.85 \acute{f}_c \beta_1 b$$

$$\bar{B} = E_s \varepsilon_u \acute{A}_s - A_s f_y + (\varepsilon_u + \varepsilon_{p_o}) E_p A_p$$

$$\bar{C} = -\varepsilon_u h A_p E_p - \varepsilon_u \acute{d} \acute{A}_s E_s$$

LOCAL FAILURE AT CUT-OFF POINT OF COMPOSITE PLATE

The interfacial shear and normal stresses in the concrete beam at the cut-off point may lead to premature local failure in the concrete beam and separation of the plate; therefore, they must be considered in design procedures. The closed form solution for the maximum shear stress at the plate end has been developed assuming linear elastic behavior of the materials, no slip, and complete composite action between the plate and concrete beam (Malek 1997; Malek et al. 1998).

$$\tau_{max} = t_p(b_3\sqrt{A} + b_2) \quad (18)$$

where t_p = thickness of the composite plate and

$$A = \frac{G_a}{t_a t_p E_p}$$

$$b_2 = \frac{\bar{y} E_p}{I_{tr} E_c}(2a_1 L_o + a_2)$$

$$b_3 = E_p \left[\frac{\bar{y}}{I_{tr} E_c}(a_1 L_o^2 + a_2 L_o + a_3) + 2b_1 \frac{t_a t_p}{G_a}\right]$$

In the above expressions, L_o is the distance between the cut-off point and the support of the beam (Fig. 7). The equation of bending moment has been assumed to be quadratic in the development of the previous equations

$$M(x_0) = a_1 x_o^2 + a_2 x_0 + a_3 \quad (19)$$

Parameters a_1, a_2, and a_3 are derived from (19); furthermore, $\bar{y}$ = distance between the composite plate and the neutral axis of the strengthened beam; I_{tr} = moment of inertia of the strengthened beam based on concrete; t_a = thickness of the adhesive layer; and G_a = shear modulus of the adhesive layer.

The maximum normal (peeling) stress is expressed by (Malek and Saadatmanesh 1996)

$$f_{n,max} = \frac{K_n}{2\beta^3}\left(\frac{V_p}{E_p I_p} - \frac{V_c + \beta M_o}{E_c I_c}\right) + \frac{q E_p I_p}{b_p E_c I_c} \quad (20)$$

where $\beta = (K_n b_p / 4 E_p I_p)^{0.25}$; and $K_n = E_a/t_a$. Furthermore, I_c and I_p = moments of inertia of the concrete and the plate beams, respectively; b_p = width of the composite plate; M_o = bending moment in the concrete beam at the location of the plate end; q = distributed load on the concrete beam; and

$$V_c = V_o - b_p \bar{y}_c t_p (b_3\sqrt{A} + b_2) \quad (21)$$

$$V_p = -b_p t_p \frac{t_p}{2}(b_3\sqrt{A} + b_2) \quad (22)$$

where V_o = shear force in the concrete beam at the location of the plate end. Eqs. (18) and (20) imply that even if the plate is extended to the point of zero moment (inflection point), there will still be nonzero shear and normal stresses at the cut-off point, which may result in local failure. However, these stresses are smaller than the case where the cut-off point is farther away from the support.

The bending moment in the concrete beam is increased by an amount, M_m, at the cut-off point of the plate, given by (Malek 1997; Malek et al. 1998)

$$M_m = L_o t_p b_p \bar{y}_c (b_3\sqrt{A} + b_2) \quad (23)$$

This moment is added to the bending moment calculated based on statistical equilibrium equations. It is noted that the load used in calculation of shear and normal stress concentrations, as well as in the calculation of increase in moment, M_m, is only the additional live load that is superimposed on the beam after strengthening, and includes the corresponding load factors. The assumption here is that some beams are strengthened while some amount of live loads could be present.

FAILURE CRITERION OF CONCRETE UNDER BIAXIAL STRESSES

At the cut-off point, the concrete beam undergoes biaxial stresses. In this case, three components of stress are present: σ_x, calculated from flexural analysis; σ_y and τ_{xy}, peeling and shear stresses calculated based on preceding discussion (Fig. 8). Only the additional live load added after strengthening is used in calculating σ_y and τ_{xy}, whereas σ_x consists of two components. The first component is obtained by considering unretrofitted beam under dead and live loads applied on the beam before upgrading. The second component is calculated using the magnified moment described above based on the additional live load. These two components are added together to obtain the total axial stress (σ_x).

The failure model for concrete under the biaxial state of stresses can be used to check the local failure of the concrete beam (Kupfer and Gerstle 1973). According to this model, the strength of concrete under different combinations of stresses is approximated by

1. Under compression-compression

$$\left(\frac{\sigma_1}{f_{cu}} + \frac{\sigma_2}{f_{cu}}\right)^2 + \frac{\sigma_1}{f_{cu}} + 3.65\frac{\sigma_2}{f_{cu}} = 0 \quad (24)$$

2. Under compression-tension

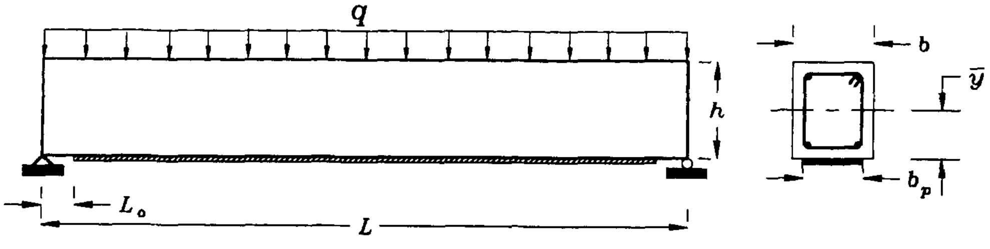

FIG. 7. View and Cross Section of Strengthened Beam

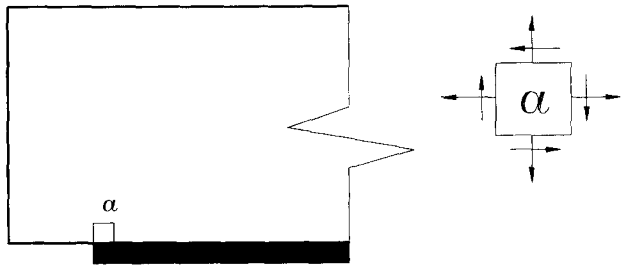

FIG. 8. Stresses Acting on Concrete Beam at Cut-Off Point of FRP Plate

$$\frac{\sigma_2}{f_{tu}} = 1 + 0.8\frac{\sigma_1}{f_{cu}} \quad (25)$$

3. Under tension-tension

$$\sigma_2 = f_{tu} = 0.295(f_{cu})^{2/3} = \text{Constant (MPa)} \quad (26a)$$

$$\sigma_2 = f_{tu} = 0.155(f_{cu})^{2/3} = \text{Constant (ksi)} \quad (26b)$$

where σ_1 and σ_2 are principal stresses in the concrete ($\sigma_1 \geq \sigma_2$), positive if tensile; f_{tu} and f_{cu} are ultimate tensile and compressive strengths of concrete, respectively. The principal stresses are calculated using stress transformation relations under plane stress conditions and are compared to the above strengths.

Shear stress concentration around flexural cracks may also lead to local debonding of the plate. Maximum shear stress in adhesive layer ignoring the minor terms is obtained from the following approximate equation (Malek 1977; Malek et al. 1998)

$$\tau_{max} = \sqrt{\frac{G_a t_p}{E_p t_a}} f_p \quad (27)$$

where f_p = axial stress in the FRP plate. The results of the analytical study and the finite-element analysis have shown very high shear stresses at the location of the cracks. Therefore, debonding around the cracks cannot be avoided and the effect of the debonded length on the nominal moment (M_n) needs to be investigated further.

The following design examples illustrate the application of the aforementioned design guidelines for strengthening of a typical RC beam, using epoxy-bonded fiber composite plate.

DESIGN EXAMPLES

Retrofit Problem

The reinforced concrete beam shown in Fig. 9 has been primarily designed for a dead load of 40 kN/m (2.74 k/ft) and a live load of 75 kN/m (5.14 k/ft). The live load on the beam is to be increased by 60% to 120 kN/m (8.22 k/ft). The service bending moment in midspan of the beam before upgrading is calculated as M_D = 180 kN/m (133 k/ft) and M_L = 337.5 kN/m (249 k/ft). Therefore, the total factored moment in the midspan of the beam is 826 kN/m (609 k/ft), which is close to flexural capacity of the RC beam, ϕM_n = 847 kN/m (625 k/ft). The factored moment in the midspan of the beam considering the superimposed live load is 1,170 kN/m (863 k/ft), which indicates the necessity of strengthening. The mechanical properties of the material used in construction of the beam as well as the composite plate are listed in Table 1.

Using an elastic analysis for the cracked section of an RC beam (without plate), the location of the neutral axis and also the moment of inertia of the transformed section based on concrete are calculated as

$$c_o = 227.2 \text{ mm } (8.95 \text{ in.}) \quad I_{tr} = 5.45 \times 10^9 \text{ mm}^4 \ (1.31 \times 10^4 \text{ in.}^4)$$

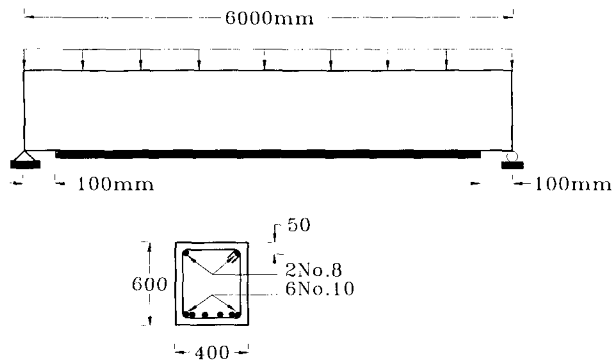

FIG. 9. Specifications of Strengthened Beam (Sample Problem)

TABLE 1. Mechanical Properties of Materials

Material properties (1)	Steel (2)	Concrete (3)	FRP (4)	Adhesive[a] (5)
E, Gpa (ksi)	200 (29,000)	27.9 (4,046)	37.23 (5,398)	2.06 (298)
f_y, MPa (ksi)	470 (68.15)	—	—	—
f_c, MPa (ksi)	—	35 (5.08)	—	—
f_u, MPa (ksi)	—	—	390 (56.55)	36 (5.22)
ε_u	—	0.003	0.0105	0.0175

[a]The reported shear modulus of adhesive was 751 MPa.

Using service loads before upgrading, tensile and compressive stresses in steel rebars and also maximum compressive stress in concrete are calculated as

$$f_s = 220 \text{ MPa} \quad f'_s = 121 \text{ MPa} \quad f_{c_o} = 21.57 \text{ MPa}$$

Using (2), the initial strain at the location of the plate is obtained as $\varepsilon_{p_o} = 1.27 \times 10^{-3}$. Using (7), it is concluded that at balanced condition the compression reinforcement will yield. Therefore, assuming β_1 = 0.8 and using (4) results in $\rho_{p,b}$ = 0.0867, $\rho_{p,max}$ = 0.065.

Using (11) and (12), $\rho_{p,bb}$ is obtained as a negative number, indicating that crushing of concrete is the predominant mode of failure. Since failure of the strengthened beam results from crushing of concrete, $\rho_{p,cy}$ is calculated as 0.0128, when using (10). Choosing a plate with nominal dimensions of t_p = 10 mm (0.4 in.), b_p = 360 mm (14.17 in.), and (ρ_p = 0.0164), and comparing ρ_p, to $\rho_{p,cy}$ shows that compression steel reinforcement yields at ultimate case. Considering crushing of concrete and also yielding of compressive reinforcement, and using (15) with corresponding parameters, c is calculated as 239 mm (9.41 in.). Next, using (16) results in the nominal moment capacity of M_n = 1,294 kN/m (954 k/ft) and design capacity ϕM_n = 1,164 kN/m (859 k/ft) close to the factored moment of 1,170 kN/m (863 k/ft). Therefore, using the selected plate is adequate from the flexural point of view.

Local Shear Failure

The analytical closed form solutions are modified for the case that the strengthened beam supports uniformly distributed loads. In this case, the maximum shear stress is calculated using the following equation:

$$\tau_{max} = \tau_o(1 + \gamma_o) \quad (28)$$

where $\tau_o = t_p E_p \bar{y} L q / 2 I_{tr} E_c$; $\gamma_o = L_o\sqrt{G_a / t_a t_p E_p}$; $\bar{y}$ = distance between the composite plate and the neutral axis of the strengthened beam based on elastic analysis of uncracked section; and L = length of the beam.

Under the same type of loading, maximum normal (peeling) stress at the cut-off point is calculated as

$$f_{n,\max} = \frac{K_n q L}{4\beta^3}\left(-\frac{\alpha t_p(1+\gamma_o)}{E_p I_p} - \frac{1 - \frac{\alpha h}{2}(1+\gamma_o) + \beta L_o}{E_c I_c}\right) \quad (29)$$

where $\alpha = b_p t_p E_p \bar{y}/I_{tr} E_c$; I_p and I_c are the moment of inertia of the isolated concrete beam and composite plate (Fig. 7), respectively, and are calculated as

$$I_p = \frac{b_p t_p^3}{12} \quad I_c = \frac{bh^3}{12}$$

Since the plate cut-off points are generally close to the inflection points, the moment of inertia, I_c, is calculated for the gross section. Bending moment in the concrete beam at the cut-off point is increased by M_m as a result of shear stress concentration at this point. The moment of this increase is calculated using the following equation:

$$M_m = \frac{\alpha L_o h L q}{4}(1 + \gamma_o) \quad (30)$$

The properties of the strengthened beam are calculated assuming uncracked section. Location of the neutral axis and moment of inertia are calculated as

$\bar{y}$ = 283 mm (11.14 in.) $I_{tr} = 9.75 \times 10^9$ mm^4 (2.34×10^4 in.^{4})

In calculating the maximum shear and normal interfacial stresses, only that part of the load that is applied after strengthening (additional live load) is taken into account. In other words

$$q = (120 - 75) \times 1.7 = 76.5 \text{ kN/m } (5.24 \text{ k/ft})$$

Replacing q, $\bar{y}$, and I_{tr} in (28), and assuming L_o = 10 cm (4 in.), results in $\tau_{\max}$ = 0.371 MPa (0.053 ksi). Parameters used in (29) are calculated as

$$K_n = 1{,}029\ \frac{\text{MPa}}{\text{mm}}\ (3{,}790 \text{ ksi/in.}),\ \beta = 0.095\ \frac{1}{\text{mm}}\ (2.413\ 1/\text{in.}),\ \gamma = 3.17$$

Consequently, the maximum normal (peeling) stress is calculated as $f_{n,\max}$ = 0.72 MPa (0.105 ksi). Using (30), the increase in bending moment is obtained as M_m = 4 kN/m (2.95 k/ft).

The bending moment in the concrete beam at the location of the plate-end due to factored loads that are applied before strengthening is 76.7 kN/m (56.6 k/ft). Therefore, the total factored bending moment of 80.7 kN/m (59.5 k/ft) is used to find the flexural stresses in the concrete beam at this point. This moment results in tensile stress of 2.26 MPa (0.328 ksi) in the concrete beam. The state of stresses acting on an element at the plate cut-off point in the concrete beam is shown in Fig. 10. Using conventional stress transformation relations maximum principal stress is calculated as 2.35 MPa (0.34 ksi). Based on (26), the ultimate tensile capacity of concrete under

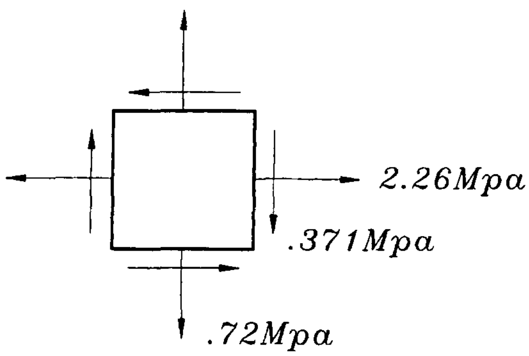

FIG. 10. Shear and Normal Stresses in Concrete at Cut-Off Point

biaxial tensile stresses is 3.15 MPa (0.456 ksi). The calculated stress of 2.35 MPa (0.34 ksi) at the cut-off point is smaller than the tensile strength of 3.15 MPa (0.456 ksi), providing an adequate margin of safety. Therefore, using the above plate with a cut-off point 10 cm (4 in.) away from the support is acceptable.

Maximum shear stress in the adhesive layer around the flexural cracks is calculated by obtaining the maximum axial stress in the FRP plate. The depth of the neutral axis and moment of inertia of the strengthened beam are calculated, assuming that the section is cracked

$\bar{y}$ = 240 mm (9.45 in.) $I_{tr} = 6.11 \times 10^9$ mm^4(1.47×10^4 in.^{4})

Axial stress in the FRP plate under the additional factored live load using the simple bending theory is 27.33 MPa (3.96 ksi). Using (27) the maximum shear stress in the adhesive layer is obtained as 8.679 MPa (1.26 ksi). This shear stress is compared with the interfacial shear strength of the adhesive, which depends on the type of the adhesive.

Repair Problem

It is assumed that in the RC beam explained in example 1, only two rebars have been used as tension reinforcement due to construction error. The beam is supposed to carry dead and live loads of 40 kN/m (2.74 k/ft) and 70 kN/m (4.8 k/ft), respectively. The additional required strength is provided by bonding a composite plate to the concrete beam. The mechanical properties of the materials were given in Table 1.

Using (7) the compression steel yields at the balanced condition; therefore, (4) is used to calculate the balanced plate ratio for yielding of the tension rebars. Based on this equation, $\rho_{p,b}$ = 0.11 and $\rho_{p,\max}$ = 0.0825. Choosing a composite plate 360 mm (14.17 in.) in width, and 6 mm (0.24 in.) in thickness provides a plate ratio of ρ_p = 0.00981, which is below the balanced ratio. Based on (11) and (12) the balanced plate ratio for verifying the mode of failure is obtained: $\rho_{p,bb}$ = 0.0103. Therefore, the rupture of the plate will be the flexural mode of failure. Considering this mode of failure, and calculating $\rho_{p,cy}$ = 0.0133, which means that compression steel will not yield at the ultimate state, the nominal bending capacity of the strengthened beam is obtained using (14) and (15): M_n = 887.1 kN/m (60.76 k/ft) and ϕM_n = 798.4 kN/m (54.69 k/ft). The maximum factored bending moment in the midspan of the beam is 778 kN/m (53.29 k/ft), which is lower than the design capacity of the strengthened beam.

Assuming that the cut-off point of the plate is 100 mm (4 in.) away from the support, and using a procedure similar to the previous example, the maximum shear and normal stresses at the cut-off point are computed as $\tau_{\max}$ = 0.741 MPa (0.107 ksi), and $f_{n,\max}$ = 1.25 MPa (0.181 ksi). The tensile stress in the concrete beam at the cut-off point considering the increase in moment is calculated as 2.06 MPa (0.299 ksi). Using these stress components in the concrete at the cut-off point, the maximum principal stress obtained is 2.47 MPa (0.358 ksi), which is lower than the tensile strength of concrete based on the biaxial model, 3.15 MPa (0.456 ksi). Therefore, the chosen plate dimensions are acceptable for this problem.

CONCLUSIONS

The design guidelines presented in this paper provide a relatively simple approach for designing concrete beams strengthened with epoxy bonded composite plates. The effect of the stresses that a concrete beam undergoes before upgrading has been considered. Rupture of the plate and crushing of concrete are the major modes of failure that are considered in calculating the ultimate strength of the plated beam. Based on the above modes of failure, the stress level in the tension and compression steel reinforcement at the ultimate state, different equations have been developed to calculate the ultimate capacity of the strengthened beam. Local failure of concrete beam at the plate end, and debonding of the plate due to shear stress concentration at the flexural cracks were also considered in developing these guidelines. Additional experimental work could be conducted to verify and refine these equations and modes of failure. Reliability studies could also be conducted to develop appropriate strength reduction factors particular to the unique nature of this structural system.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$\bar{A}$ = parameter used in calculating depth of neutral axis;
$\bar{B}$ = parameter used in calculating depth of neutral axis;
$\bar{C}$ = parameter used in calculating depth of neutral axis;
c = depth of neutral axis after strengthening;
c_o = depth of neutral axis before strengthening;
d = depth of tensile steel reinforcement;
d' = depth of compressive steel reinforcement;
E_c = modulus of elasticity of concrete;
E_p = modulus of elasticity of composite plate;
E_s = modulus of elasticity of steel;
f_{cu} = compressive strength of concrete;
f'_c = compressive strength of concrete;
f_p = axial stress in composite plate;
f_{pr} = tensile strength of composite plate;
f_{tu} = tensile strength of concrete;
f_y = yield stress of steel;
G_a = shear modulus of adhesive;
h = height of beam;
I_c = moment of inertia of concrete beam;
I_p = moment of inertia of plate;
I_{tr} = moment of inertia of transformed section;
I_{tro} = moment of inertia of transformed section before strengthening;
K_n = parameter used in calculating peeling stress;
L = length of beam;
L_o = distance between cut-off point and support;
M_m = magnifying moment;
M_n = nominal flexural strength of strengthened beam;
M_o = bending moment before strengthening;
q = uniformly distributed load;
t_a = thickness of adhesive;
$\bar{y}$ = distance between plate and neutral axis of strengthened beam;
α = parameter used in calculating peeling stress;
β = parameter used in calculating peeling stress;
β_1 = parameter of rectangular stress block;
γ_o = parameter used in calculating maximum shear stress;
ε_p = strain in composite plate;
ε_{p_o} = strain in location of composite plate before strengthening;
ε_r = tensile strain in composite plate at rupture;
ε'_s = strain in compressive steel reinforcement;
ε_u = ultimate strain of concrete;
η_1 = parameter used in equation of balanced plate ratio;
η_2 = parameter used in equation of balanced plate ratio;
η_3 = parameter used in equation of balanced plate ratio;
ρ = tensile steel ratio;
ρ' = compressive steel ratio;
ρ_p = plate ratio;
$\rho_{p,b}$ = plate ratio at balanced conditions;
$\rho_{p,cy}$ = minimum plate ratio for yielding of compression steel;
$\rho_{p,\max}$ = maximum acceptable plate ratio;
σ_1 = maximum principle stress;
σ_2 = minimum principle stress;
τ_o = parameter used in calculating maximum shear stress; and
$\tau_{\max}$ = maximum shear stress.