A New Punching Shear Equation for Two-Way Concrete Slabs Reinforced with FRP Bars

by S. El-Gamal, E.F. El-Salakawy, and B. Benmokrane

Synopsis: Recently, there has been a rapid increase in using the non-corrodible fiber-reinforced polymers (FRP) reinforcing bars as alternative reinforcements for concrete structures especially those in harsh environments. The elastic stiffness, ultimate strength, and bond characteristics of FRP reinforcing bars are quite different from those of steel, which affect the shear capacity. The recently published FRP design codes and guidelines include equations for shear design of one-way flexural members. However, very little work was done to investigate the punching shear behavior of two-way slabs reinforced with FRP bars. The current design provisions for shear in two-way slabs are based on testing carried out on steel reinforced slabs. This study presents a new model to predict shear capacity of two-way concrete slabs that were developed based on extensive experimental work. The accuracy of this prediction model was evaluated against the existing test data. Compared to the available design models, the proposed shear model seems to have very good agreement with test results with better predictions for both FRP and steel-reinforced concrete two-way slabs.

Keywords: concrete slabs; design models; FRP reinforcement; punching shear

Sherif El-Gamal is a Ph.D. Candidate in the Department of Civil Engineering at the Université de Sherbrooke, Canada. He received his BSc and MSc in structural engineering from the Menoufiya University, Egypt. His research interests include large-scale experimental testing and finite element modeling of concrete structures reinforced with steel and FRP composites.

ACI member **Ehab El-Salakawy** is a Research Associate Professor in the Department of Civil Engineering at the Université de Sherbrooke, Canada. His research interests include large-scale experimental testing and finite element modeling of reinforced concrete structures, construction, and rehabilitation of concrete structures reinforced with FRP composites. He has been involved in the design, construction, testing, and monitoring of several FRP-reinforced concrete bridges in North America.

ACI member **Brahim Benmokrane** is an NSERC Research Chair Professor in FRP Reinforcement for Concrete Structures in the Department of Civil Engineering at the Université de Sherbrooke, Sherbrooke, Québec, Canada. He is a project leader in ISIS Canada Network of Centers of Excellence. His research interests include the application and durability of advanced composite materials in civil engineering structures and structural health monitoring with fiber optic sensors. He is a member in the CSA (Canadian Standard Association) committees on FRP structural components and FRP reinforcing materials for buildings (S806) and bridges (S6), and ACI committee 440 'FRP Reinforcement.

INTRODUCTION

The long-term durability of reinforced concrete structures has become a major concern in the construction industry. One of the main factors reducing durability and service life of the reinforced concrete structures is the corrosion of steel reinforcement. Many steel-reinforced concrete structures exposed to de-icing salts and marine environment require extensive and expensive maintenance. Recently, the use of fiber-reinforced polymer (FRP) as alternative reinforcing material in reinforced concrete structures has emerged as innovative solution to the corrosion problem. In addition to the non-corrosive nature of FRP materials, they also have a high strength-to-weight ratio which makes them attractive as reinforcement for concrete structures.

Extensive research programs have been conducted to investigate the flexural behavior of concrete members reinforced with FRP reinforcement. On the other hand, the shear behavior of concrete members in general and especially punching shear of two-way slabs, reinforced with FRP bars has not yet been fully explored. Since early 1960s, much research has been carried out on punching shear behavior of slabs reinforced with conventional steel and several design models were proposed (Moe 1961; kinnunen and Nylander 1960; Vanderbilt 1972; Hewitt and Batchelor 1975, El-Salakawy et al. 1999 and 2000). However, these models can not be directly applied to FRP-reinforced concrete slabs due to the difference in mechanical properties between FRP and steel reinforcement. The modulus of elasticity for the commercially available glass and aramid FRP bars is 20 to 25 % that of steel compared to 60 to 75 % for carbon FRP bars.

Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This results in the development of wider and deeper cracks. Deeper cracks decrease the contribution to shear strength from the uncracked concrete due to the lower depth of concrete in compression. Wider cracks, in turn, decrease the contributions from aggregate interlock and residual tensile stresses. Additionally, due to the relatively small transverse strength of FRP bars and relatively wider cracks, the contribution of dowel action may be negligible.

PUNCHING SHEAR PROVISIONS FOR FRP-REINFORCED CONCRETE TWO-WAY SLABS

For steel reinforced concrete slabs, due to the relatively high modulus of elasticity of steel, the dominant factor determining the concrete shear resistance will be the area of concrete in the compression zone (after cracking), which remains practically unchanged as the depth to the neutral axis does not vary much. This is true when using common steel reinforcement ratios. Therefore, most design codes do not imply the flexural reinforcement ratio in determining the punching shear capacity of steel reinforced concrete slabs. However, due to the relatively low modulus of elasticity of FRP, the concrete shear strength of FRP-reinforced concrete slabs becomes more sensitive to the reinforcement stiffness as the depth to neutral axis is reduced significantly after cracking. Thus, it deemed necessary to investigate the behavior and determine the punching shear capacity of concrete slabs reinforced with FRP bars.

Current recommendations in the American Concrete Institute Code, ACI 318-05 (2005), and the British Standards, BS 8110-97 (1997), were empirically derived for slabs reinforced with steel reinforcement. There is a lack of design and prediction models related to the punching shear strength of concrete slabs reinforced with FRP composite bars. El-Ghandour et al. (1999) conducted tests on FRP-reinforced concrete slabs with and without shear reinforcement. They investigated both the strain and stress approaches which used to determine the punching shear capacity of FRP reinforced concrete slabs by converting the actual FRP reinforced area to an equivalent steel area to be used in equations derived for steel reinforced slabs. They concluded that strain approach represents a lower limit and the stress approach represents an upper limit for punching capacity. They introduced a modified approach which incorporates both the strain and stress approaches as a modification for the FRP area to equivalent steel area used in the BS-8110 (British Standard Institution 1997)

In addition, they conducted experimental tests on FRP-reinforced concrete flat slabs and compared the ultimate capacity with that predicted by different codes. They also suggested a modification to the ACI-318-95 (1995) equation through multiplying the obtained shear strength value by the term $(E_{FRP}/E_{steel})^{1/3}$ as follow:

$$V_{c,\,EL} = 0.33\sqrt{f'_c}\left(\frac{E_{FRP}}{E_s}\right)^{1/3} b_o\, d \tag{1}$$

where f'_c is the specified compressive strength of concrete (MPa), E_{FRP} and E_s are the modulus of elasticity of FRP and steel respectively (GPa), b_o is the critical perimeter at a distance of $d/2$ away from the column face (mm), and d is the average flexural depth of the slab (mm). Test results showed that this modification leads to accurate predictions of the punching capacity of their tested FRP reinforced slabs without shear reinforcement.

Matthys and Taerwe (2000) studied the punching shear behavior of concrete slabs reinforced with different types of FRP grids. Test results showed that for FRP-reinforced slabs with similar flexural strength as the steel reinforced control slab, the obtained punching load and stiffness were less. They also concluded that increasing the slab depth enhanced cracking behavior, ultimate load, and stiffness in the fully cracked state. They noticed that, as the elastic stiffness (ρE) of the tensile FRP reinforcing mat increases, the punching capacity increases and the slab deflection at ultimate capacity decreases. From test results, they suggested a modification to BS-8110 (British Standard Institution 1997) to account for the use of FRP bars when determining the punching shear capacity as:

$$V_{c,\,MT} = 1.36\frac{\left(100\rho_f \dfrac{E_{FRP}}{E_s} f'_c\right)^{1/3}}{d^{1/4}} b_o\, d \tag{2}$$

where ρ_f is the reinforcement ratio of the tensile FRP mat, b_o is the rectangular or square control perimeter at a distance of 1.5 d away from the loaded area (mm). The remaining parameters are as defined in Eq. 1.

Ospina et al. (2003) examined the punching shear behavior of flat slabs reinforced with FRP bars or grids. Four full scale square slab specimens (2150 × 2150 × 155 mm) were built and tested under central concentrated load in static punching. The main variables were the reinforcement materials, steel and GFRP, the type of FRP reinforcing mat, bars or grids, and the reinforcement ratio. Test results showed that slabs reinforced with FRP reinforcement display the same kinematics features observed by Kinnunen and Nylander (1960) for steel reinforcement. The behavior was affected by the elastic stiffness (ρE) of the tensile reinforcing mat and bond with concrete. They introduced an incremental modification to the equation presented by Matthy and Taerwe (2000) as:

$$V_{c,\,Osp.} = 2.77\left(\rho_f f'_c\right)^{1/3}\sqrt{\frac{E_{FRP}}{E_s}}\; b_o d \tag{3}$$

where all parameters are as defined in Eq. 2

The sub-committee ACI440H is currently considering the introduction of a new provision to account for the punching capacity of two-way concrete slabs reinforced with

FRP bars in the next edition of the ACI-440.1R guide. This sub-committee has proposed the use of Eq. (4). This equation considers the effect of reinforcement stiffness to account for the shear transfer in two-way concrete slabs.

$$V_c = \frac{4}{5}\sqrt{f'_c}\, b_o\, c \tag{4}$$

where b_o is the perimeter of critical section for slabs and footings (mm) and c is the cracked transformed section neutral axis depth (mm), $c = kd$.

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f \tag{4a}$$

In the evaluation of Eq. (4), b_o should be evaluated at $d/2$ away from the column face. In addition, the shape of the critical surface should be the same as that of the column. Equation (4) can be rewritten as Eq. (5). This equation is simply the basic ACI 318-02 (2002) concentric punching shear equation for steel-reinforced slabs, V_c, modified by the factor $\left(\frac{5k}{2}\right)$ which accounts for the axial stiffness of the FRP reinforcement.

$$V_c = \left(\frac{5k}{2}\right) 0.33\sqrt{f'_c}\, b_o\, d \tag{5}$$

PROPOSED DESIGN EQUATION

An extensive research program has been carried out by the authors at the University of Sherbrooke to investigate the punching shear capacity of bridge deck slabs reinforced with FRP bars Through this program, eight full size bridge deck slab prototypes (3000 × 2500 × 200 mm) were constructed and tested to failure in the laboratory. The test parameters were the type and ratio of the bottom reinforcement in the transverse direction. The used FRP bars were manufactured by combining the pultrusion process and an in-line coating process for the outside sand surface (Pultrall Inc., Thetford Mines, Québec, Canada). The GFRP bar was made from high-strength E-glass fibers (75% fiber by volume) with a vinyl ester resin, additives, and fillers. The carbon FRP (CFRP) bar was made of 73% carbon fiber by volume, a vinyl ester resin, additives and fillers. All tested slabs had the same GFRP reinforcement in all directions except in the bottom transverse direction. Five slabs were reinforced with GFRP bars (1 to 2%) and two slabs were reinforced with CFRP bars (0.35 and 0.69%). The remaining slab was reinforced with steel bars (0.3%) in all direction as a control slab. The axial stiffness of the bottom transverse reinforcement of the tested slabs ranged between 415 and 830 N/mm^2. The slabs were supported on two steel girders spaced at 2000-mm centre-to-centre and were subjected to a monotonic single concentrated load over a contact area of 600 × 250 mm to simulate the foot print of sustained truck wheel load (CL-625 Truck as specified by the Canadian Highway Bridge Design Code, CSA-S6-00 2000) acting on the centre of the slab. The steel girders were supported over a span of 3000 mm in the longitudinal direction. The concrete slab was bolted to the supporting steel girders through holes in the slabs. In addition, three steel cross frames were used to prevent the two girders from lateral movement. Figure 1 shows a Photo of the test set-up. All the tested slabs failed by punching shear as shown in Figure 2. More details on the test set-up and results can be found elsewhere (El-Gamal et al. 2004). Based on this work, a new model to predict the punching shear capacity of concrete deck slabs reinforced with FRP bars was proposed. This model takes into consideration the most important factors to give better agreement with the experimental results and is given by Eq. 6.

$$V_c = V_{c,ACI\ 318} \times \alpha \times (1.2)^N \tag{6}$$

$$V_c = 0.33\sqrt{f_c'}\ b_o\ d \times \alpha \times (1.2)^N \tag{6a}$$

Equation 6 is a modified form of ACI 318-05 equation (11-35) that calculates the punching shear capacities of concrete slabs reinforced with conventional steel. As all designers and engineers are familiar with the ACI 318 equation, it was intended to keep the equation in the same form and introduce two new parameters α and N to take into account the effects of the axial stiffness of bottom main reinforcement and the continuity of the slab where:

f_c' = the specified compressive strength of concrete (MPa),
b_o = the rectangular critical perimeter at a distance of *d/2* away from the loaded area (mm),
d = the effective depth of the slab (mm);

N is the continuity factor taken as,
= 0 (for one panel slabs);
= 1 (for slab continuous along one axis);
= 2 (for slabs continuous along their two axes);

α is a new parameter which is a function of the flexural stiffness of the tensile reinforcement (ρE), the perimeter of the applied load, and the effective depth of the slab was introduced.

$$\alpha = 0.5\,(\rho\,E)^{1/3}\left(1 + {}^{8d}\!/_{b_o}\right) \tag{6b}$$

where ρ and E are the reinforcement ratio and modulus of elasticity (in GPa) of the main bottom reinforcement, respectively.

This model can be used to predict the punching shear capacity of concrete two-way slabs reinforced with either FRP or steel reinforcement. Since the results used in this study were based on punching shear tests carried out on isolated specimens (simply supported along the lines of contra-flexure around the load area), the continuity factor can be removed ($N = 0$) and the proposed equation will be reduce to the following: .

$$V_{c,\Pr oposed} = V_{c,ACI\ 318} \times \alpha = 0.33\sqrt{f_c'}\ b_o\ d \times \alpha \tag{6c}$$

The term α can be calculated according Eq. (6b) where ρ is the average ratio of tensile reinforcement in both directions.

This proposed model can be generally applied to predict the punching shear capacity of two-way concrete slabs reinforced with FRP or steel reinforcement. Equation (6) was verified against available experimental data conducted by other researchers (Ahmed et al 1993; Banthia et al. 1995; El-Ghandour et al. 1999; Matthys and Taerwe 2000; Ospina et al. 2003; Hussein and Rashid 2004) and very good agreement with the test results was obtained as shown in Figure 3. In this paper, Eq. (4) was selected to be compared to the proposed Eq. (6). Equation (4) is the most recent design model that was based on a comprehensive study performed by Tureyen and Frosch (2003) investigating the shear strength of FRP and steel reinforced concrete members including the data used in this paper. Tureyen and Frosch (2003) concluded that the above presented models (Eq. 1 to 5), except Eq. (4), give either inconsistent or over conservative predictions compared to the experimental results. They recommended the use of Eq. (4), which is currently under consideration to be included in the next edition of the ACI 440.1R.

Table 1 compares the predictions of Eq. (4) with those of the proposed equation. It can be noted that the proposed equation (6) gives very good agreement with test results, yet conservative. For Eq. (4), the mean value of the experimental/prediction was 2.64, with a standard deviation of 0.64 (coefficient of variation of 24.4%). While the corresponding values for the proposed model were 1.34 and 0.17 (coefficient of variation of 12.9%). The large standard deviation of equation (4) means that it does not reflect well the functional relationship between the shear parameters and the shear strength. The larger scatter of Eq. (4) results as shown in Fig. 2 cannot be rectified by adjusting the multiplier 4/5.

Furthermore, the proposed model was compared to two-way slabs reinforced with steel reinforcement (Marzouk and Hussein 1991; Elstner and Hognested 1960; Kinnunen and Nylander, 1960; Emam 1995; Hognested 1964). The predictions were also in good agreement with the test results as shown in Figure 4. In addition, the predictions of Eq. (4) with those of the proposed equation were compared as listed in Table 2. Again, better agreement with test results was obtained by the proposed equation. This suggests that the proposed equation can be used to predict the punching shear capacities of both FRP and steel-reinforced concrete two-way slabs. The average of the overall test/predicted values of the tests (slabs reinforced with steel and FRP) was 1.21, the standard deviation was 0.17, and the coefficient of variation was 14.3%.

CONCLUSIONS AND RECOMMENDATIONS

This paper presents a new equation to predict the punching shear strength of two-way concrete slabs reinforced with FRP reinforcement (bars and grids). In the proposed model, a new parameter, α, which is a function of the flexural stiffness of the tensile reinforcement (ρE), the perimeter of the applied load, and the effective depth of the slab was introduced to the original ACI 318-05 (2005), Eq. 11-35, for conventional steel. In addition, a summary of all available punching shear design models for FRP-reinforced slabs were also presented. The proposed design model (Eq. 6c) and Eq. 4 (under consideration by the sub-committee ACI 440H) were compared to most of, if not all, the published test results on FRP-reinforced concrete two-way slabs. Based on the performed study, the following conclusions can be drawn:

1. The proposed design model, Eq. (6), gives very good agreement with test results of FRP-reinforced concrete slabs, yet conservative.
2. The proposed model gives better predictions compared to (Eq. 4). For Eq. (4), the mean value of the experimental/prediction was 2.64, with a standard deviation of 0.64 (coefficient of variation of 24.4%). While the corresponding values for the proposed model were 1.34 and 0.17 (coefficient of variation of 12.9%).
3. This model can be generally applied to predict the punching shear capacity of two-way concrete slabs reinforced with FRP or steel reinforcement.

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Table 1 — Comparison of the proposed (Eq. 7) and the ACI 440-H (Eq. 4) models for FRP-reinforced slabs.

Reference	Slab	Column size* (mm)	d (mm)	f'_c MPa	ρ_f (%)	E_f (GPa)	α	$V_{c,test}$ (kN)	$V_{c,ACI\ 440H}$ (kN)	$V_{c,prop.}$ (kN)	$\frac{V_{c,test}}{V_{c,ACI\ 440H}}$	$\frac{V_{c,test}}{V_{c,prop.}}$
Ahmed, Zia, Yu & Xie (1993)	SN1	S 75	61	42.4	0.95	113.0	0.97	93	39.9	69.3	2.33	1.34
	SN2	S 75	61	44.6	0.95	113.0	0.97	78	40.5	71.0	1.93	1.10
	SN3	S 100	61	39	0.95	113.0	0.90	96	46.1	72.9	2.08	1.32
	SN4	S 100	61	36.6	0.95	113.0	0.90	99	45.3	70.6	2.18	1.40
Banthia, Al-Asaly & Ma (1995)	I	C 100	55	41.0	0.31	100.0	0.58	65	18.2	41.7	3.57	1.56
	II	C 100	55	52.9	0.31	100.0	0.58	61	19.5	47.4	3.13	1.29
El-Ghandour, Pilakoutas & Waldron (1999)	SG1	S 200	142	33.3	0.22	45.0	0.42	170	73.1	156.6	2.33	1.09
	SC1	S 200	142	34.7	0.18	110.0	0.53	229	102.6	201.4	2.23	1.14
	SG2	S 200	142	46.6	0.47	45.0	0.55	271	114.4	238.6	2.37	1.14
	SG3	S 200	142	30.3	0.47	45.0	0.55	237	102.1	192.4	2.32	1.23
	SC2	S 200	142	29.6	0.43	110.0	0.71	317	147.0	248.7	2.16	1.27
Matthys & Taerwe (2000) (continued)	C1	C 150	96	36.7	0.26	91.8	0.55	181	43.4	104.3	4.17	1.74
	C1’	C 230	96	37.3	0.26	91.8	0.49	189	57.8	124.3	3.27	1.52
	C2	C 150	95	35.7	1.05	95.0	0.89	255	81.2	162.8	3.14	1.57
	C2’	C 230	95	36.3	1.05	95.0	0.79	273	108.2	194.4	2.52	1.40
	C3	C 150	126	33.8	0.52	92.0	0.75	347	86.2	199.7	4.03	1.74
	C3’	C 230	126	34.3	0.52	92.0	0.67	343	111.6	231.6	3.07	1.48
	CS	C 150	95	32.6	0.19	147.6	0.58	142	44.7	101.9	3.18	1.39
	CS’	C 230	95	33.2	0.19	147.6	0.52	150	59.6	121.8	2.52	1.23
	H1	C 150	95	118.0	0.64	37.3	0.55	207	58.2	183.8	3.56	1.13

Table 1 (Continued) — Comparison of the proposed (Eq. 7) and the ACI 440-H (Eq. 4) models for FRP-reinforced slabs.

Reference	Slab	Column size* (mm)	d (mm)	f'_c MPa	ρ_f (%)	E_f (GPa)	α	$V_{c.\,test}$ (kN)	$V_{c,\,ACI\,440\,H}$ (kN)	$V_{c,\,prop.}$ (kN)	$\frac{V_{c,\,test}}{V_{c,\,ACI\,440\,H}}$	$\frac{V_{c,\,test}}{V_{c,\,prop.}}$
(continued) Matthys & Taerwe (2000)	H2	C 150	89	35.8	3.78	40.7	1.01	231	89.3	169.2	2.59	1.37
	H2’	C 80	89	35.9	3.78	40.7	1.19	171	63.2	140.9	2.71	1.21
	H3	C 150	122	32.1	1.21	44.8	0.77	237	85.8	191.9	2.76	1.23
	H3’	C 80	122	32.1	1.21	44.8	0.90	217	63.7	165.9	3.41	1.31
Ospina, Alexander & Cheng (2003)	GFR1	S 250	120	29.5	0.73	34.0	0.52	217	99.9	164.9	2.17	1.32
	GFR2	S 250	120	28.9	1.46	34.0	0.65	260	136.5	205.6	1.90	1.26
	NEF1	S 250	120	37.5	0.87	28.4	0.52	206	106.3	185.6	1.94	1.11
Hussein & Rashid (2004)	G-S1	S 250	100	40.0	1.18	42.0	0.62	249	117.5	181.7	2.12	1.37
	G-S2	S 250	100	35.0	1.05	42.0	0.60	218	107.5	163.5	2.03	1.33
	G-S3	S 250	100	29.0	1.67	42.0	0.70	240	125.7	173.7	1.91	1.38
	G-S4	S 250	100	26.0	0.95	42.0	0.58	210	94.7	136.3	2.22	1.54
										Mean	2.64	1.34
										σ	0.64	0.17
										Co.V. (%)	24.37	12.87

* C refers to circular columns
S refers to square columns

Table 2 — Comparison of the proposed (Eq. 7) and the ACI 440-H (Eq. 4) models for steel-reinforced slabs.

Reference	Slab	Column size* (mm)	d (mm)	f'_c (MPa)	ρ_f (%)	E_f (GPa)	α	$V_{c,test}$ (kN)	$V_{c,ACI 440 H}$ (kN)	$V_{c,prop.}$ (kN)	$\frac{V_{c,test}}{V_{c,ACI 440 H}}$	$\frac{V_{c,test}}{V_{c,prop.}}$
Elstner and Hognestad (1956)	A-1a	S254	117	14.1	1.15	200	1.08	302	206.0	231.6	1.47	1.30
	A-1b	S254	117	25.2	1.15	200	1.08	365	246.3	309.6	1.48	1.18
	A-1c	S254	117	29.0	1.15	200	1.08	356	257.0	332.1	1.39	1.07
	A-1d	S254	117	36.8	1.15	200	1.08	352	276.1	374.1	1.28	0.94
	A-1e	S254	117	20.2	1.15	200	1.08	356	230.2	277.2	1.55	1.28
	A-2a	S254	114	13.7	2.47	200	1.38	334	258.2	282.7	1.29	1.18
	A-2b	S254	114	19.5	2.47	200	1.38	400	290.5	337.3	1.38	1.19
	A-2c	S254	114	37.5	2.47	200	1.38	467	359.4	467.7	1.30	1.00
	A-7b	S254	114	28.0	2.47	200	1.38	512	327.1	404.1	1.57	1.27
	A-3a	S254	114	12.8	3.70	200	1.58	356	285.5	312.6	1.25	1.14
	A-3b	S254	114	22.6	3.70	200	1.58	445	348.3	415.4	1.28	1.07
	A-3c	S254	114	26.6	3.70	200	1.58	534	368.2	450.7	1.45	1.18
	A-4	S356	117	26.2	1.15	200	0.99	400	317.8	368.9	1.26	1.08
	A-5	S356	114	27.8	2.47	200	1.26	534	416.8	471.6	1.28	1.13
	B-9	S254	114	43.9	2.00	200	1.28	504	349.4	473.2	1.44	1.07
	B-10	S254	114	13.5	3.00	200	1.47	330	273.2	300.4	1.21	1.10
	B-11	S254	114	50.5	3.00	200	1.47	579	423.9	581.0	1.37	1.00

Table 2 (Continued) — Comparison of the proposed (Eq. 7) and the ACI 440-H (Eq. 4) models for steel-reinforced slabs.

Reference	Slab	Column size* (mm)	d (mm)	f'_c (MPa)	ρ_f (%)	E_f (GPa)	α	$V_{c,test}$ (kN)	$V_{c, ACI 440 H}$ (kN)	$V_{c,prop.}$ (kN)	$\frac{V_{c,test}}{V_{c, ACI 440 H}}$	$\frac{V_{c,test}}{V_{c,prop.}}$
Kinnunen and Nylander (1960)	05	C150	117	28.5	0.80	200	1.10	255	158.9	241.6	1.61	1.06
	06	C150	118	27.8	0.79	200	1.10	275	158.8	241.0	1.73	1.14
	24	C150	128	28.0	1.01	200	1.21	430	197.9	301.8	2.17	1.42
	25	C150	124	26.7	1.04	200	1.22	408	188.5	281.8	2.16	1.45
	32	C150	123	28.0	0.49	200	0.94	258	138.2	221.5	1.87	1.17
	33	C150	125	28.3	0.48	200	0.94	258	140.6	227.3	1.84	1.14
Hognestad et al. (1964)	H1L3	S250	114	30.0	1.10	200	1.06	312	243.9	317.3	1.28	0.98
	H1L4	S250	114	27.5	1.10	200	1.06	312	237.5	303.8	1.31	1.03
Marzouk and Hussein (1991) (continued)	NS1	S150	95	42.0	1.47	200	1.27	320	169.8	253.2	1.88	1.26
	HS2	S150	95	70.0	0.84	200	1.06	249	156.8	271.3	1.59	0.92
	HS7	S150	95	74.0	1.19	200	1.19	356	184.5	313.3	1.93	1.14
	HS3	S150	95	69.0	1.47	200	1.27	356	197.1	324.6	1.81	1.10
	HS4	S150	90	66.0	2.37	200	1.47	418	217.8	340.5	1.92	1.23
	NS2	S150	120	30.0	0.94	200	1.17	396	178.7	273.1	2.22	1.45
	HS5	S150	125	68.0	0.64	200	1.04	365	204.1	387.8	1.79	0.94
	HS6	S150	120	70.0	0.94	200	1.17	489	228.9	417.1	2.14	1.17
	HS8	S150	120	69.0	1.11	200	1.23	436	244.5	437.7	1.78	1.00
	HS9	S150	120	74.0	1.61	200	1.39	543	290.7	513.1	1.87	1.06
	HS10	S150	120	80.0	2.33	200	1.58	645	344.4	603.4	1.87	1.07
	HS11	S150	70	70.0	0.95	200	1.01	196	109.3	172.4	1.79	1.14

Table 2 (Continued) — Comparison of the proposed (Eq. 7) and the ACI 440-H (Eq. 4) models for steel-reinforced slabs.

Reference	Slab	Column size* (mm)	d (mm)	f'_c (MPa)	ρ_f (%)	E_f (GPa)	α	$V_{c,test}$ (kN)	$V_{c,ACI\ 440\ H}$ (kN)	$V_{c,prop.}$ (kN)	$V_{c,test}/V_{c,ACI\ 440\ H}$	$V_{c,test}/V_{c,prop.}$
continued) Marzouk and Hussein (1991)	HS12	S150	70	75.0	1.52	200	1.19	258	135.5	208.7	1.90	1.24
	HS13	S150	70	68.0	2.00	200	1.30	267	146.8	217.7	1.82	1.23
	HS14	S220	95	72.0	1.47	200	1.15	498	256.7	384.9	1.94	1.29
	HS15	S300	95	71.0	1.47	200	1.06	560	320.6	442.8	1.75	1.26
Emam (1995)	02	S250	120	37.0	1.00	200	1.04	489	267.3	370.3	1.83	1.32
	08	S250	120	67.0	1.00	200	1.04	511	317.9	498.3	1.61	1.03
										Mean	1.64	1.15
										σ	0.29	0.13
										Co.V. (%)	17.89	11.59

* C refers to circular columns
S refers to square columns

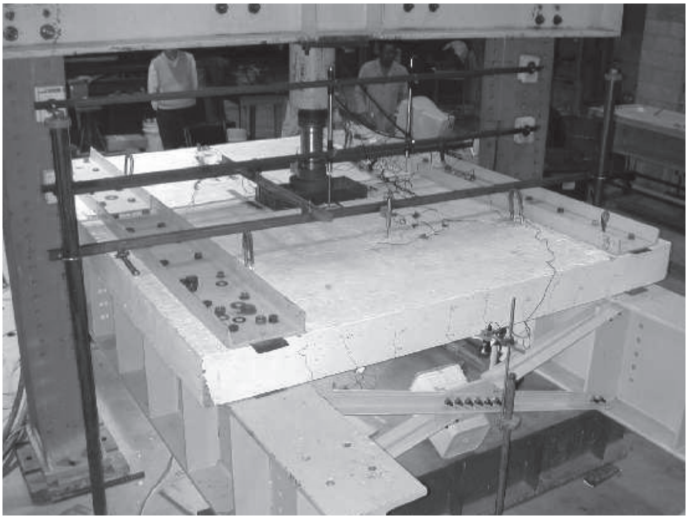

Fig. 1— Test set-up [El-Gamal et al. 2004].

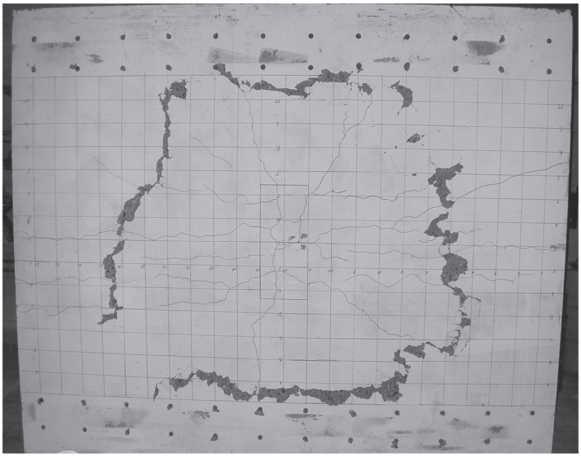

Fig. 2— Typical punching shear failure [El-Gamal et al. 2004].

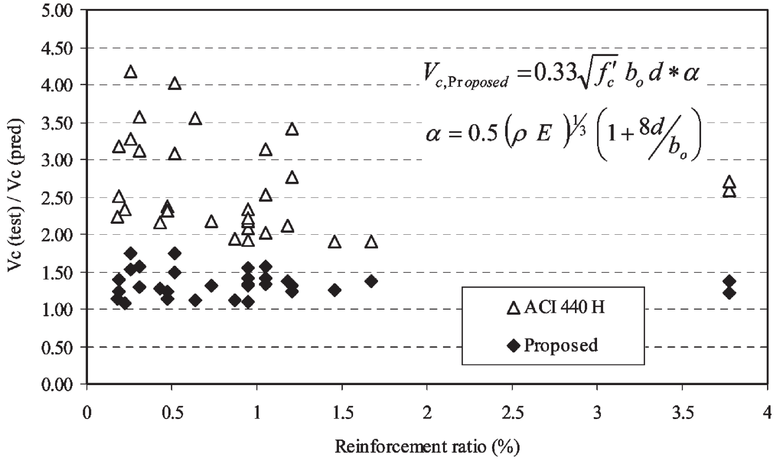

Fig. 3 — Comparison of the proposed and the ACI 440-H models against experimental results for FRP-reinforced concrete two-way slabs.

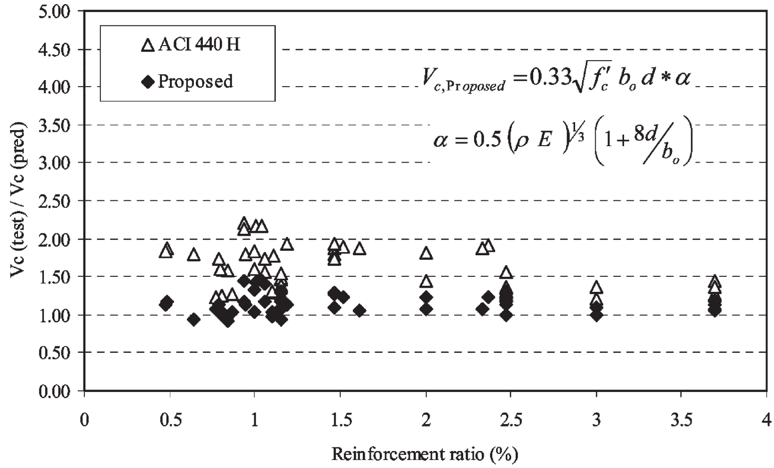

Fig. 4—Comparison of the proposed and the ACI 440-H models against experimental results for steel-reinforced concrete two-way slabs.