A GENERAL METHOD FOR DEFLECTIONS EVALUATION OF FIBER REINFORCED POLYMER (FRP) REINFORCED CONCRETE MEMBERS

Maria Antonietta Aiello and Luciano Ombres, University of Lecce, Lecce, Italy

Abstract

A general method for calculating deflections of FRP reinforced concrete members was used in the paper. The method is found based on the use of moment-curvature diagrams of a concrete block between two contiguous cracks. The local bond-slip and the tension stiffening effects were considered. An experimental investigation, carried out on concrete beams reinforced with Carbon FRP rebars was described and obtained results were compared with predictions of the proposed model and those of models usually adopted in the analysis of FRP reinforced concrete beams.

Introduction

The analysis of cracking and deformability, under service conditions, is the most critical aspect when Fiber Reinforced Polymer (FRP) rebars are used as reinforcement of flexural concrete members. The low elastic modulus of FRP rebars, in fact, involves high deformability and high cracks width as well as a lack of ductility if compared with traditional steel reinforced concrete members. The serviceability conditions (deflection and cracking), rather than strength, become, then, critical for the design of FRP reinforced concrete members.

Cracking and deflections of reinforced concrete members are strongly influenced by the bond properties of the reinforcing rebars. However, the bond performance of FRP rebars varies within a wide range because of several products are commercially available and characterized from various outer surface treatment aiming to obtain bond properties comparable with those of steel rebars.

Others parameters influencing cracking and deflection of concrete beams are the tension stiffening effects (that is the tension carried by the concrete between two cracked sections), the concrete strength, the FRP strength, the elastic modulus of the FRP rebars and the rebars diameter. All those parameters have to be considered for calculating deflections of FRP reinforced concrete beams.

Currently some models have been proposed for deflection calculation of FRP reinforced flexural concrete elements; however their general validity is not completely proved giving in some cases significant scatters with respect to experimental results. The mentioned models were defined re-using those proposed for steel reinforced concrete structures with an appropriate calibration of involved coefficients to take into account the specific properties of FRP rebars.

Modified relationships of the effective moment of inertia J_e are proposed from Codes (ACI, CAN, JSCE) to calculate the deflection of cracked GFRP (Glass FRP) reinforced concrete structures. Deflection values are determined by means of equations derived from linear elastic analysis considering J_e as uniform moment of inertia throughout the beam length. The evaluation of the effective moment of inertia, as well known, is based on empirical considerations; its applicability to the analysis of beams is correct only for certain loading and supporting conditions.

A correct calculation of beam deflections for any load and boundary conditions have to consider the moment-curvature law and the use of the principle of the virtual work.

Recently a method for calculating deflection by using the moment–curvature law has been proposed (Razaqpur and oth., 2000); in this model a trilinear moment-curvature law has been adopted supposing that tension stiffening is negligible and the variation of the neutral axis position after cracking is relatively small and has negligible effects on the curvature of the member under service loads.

A more general model, founded on a block model, has been proposed from Authors to investigate the flexural behavior of FRP reinforced concrete structures. A non-linear procedure which considers different cracking configurations, derived from a cracking analysis based on slip and bond stresses, was adopted for evaluating cracks width, cracks spacing and deflections. The procedure allows taking into account both the bond-slip law between FRP rods and concrete and the contribution of the tension stiffening between two adjacent cracks. (Aiello and Ombres, 1997; 1999; 2000; 2001).

The model is adopted for the analysis described in this paper; further indications about the general procedure are given and a comparison between deflection values furnished by the general model, Code models, theoretical models available in the literature and experimental results obtained by flexural tests on Aramid FRP (AFRP) and Carbon FRP (CFRP) reinforced concrete beams is presented and discussed.

The analysis of obtained results allows checking the effectiveness of the different approaches used for calculating deflections of FRP reinforced concrete structures; besides, some indications useful for a design point of view are pointed out and discussed.

Current Models of Deflection Calculation of FRP Reinforced Concrete Beams

As previously stated, models used for calculating deflections of FRP reinforced concrete beams can be grouped as empirical models and rational models; the first ones are founded on the use of an effective moment of inertia empirically determined on the basis of experimental results while the second ones are founded on the use of the moment-curvature law.

Empirical models

In such models deflections calculation is mainly based on equations derived from linear elastic analysis assuming the effective moment of inertia to be uniform along the beam. The general expression of the effective moment of inertia is

$$J_e = \beta J_1 (1-\zeta) + \zeta J_2$$

being J_1 and J_2 the moment of inertia of sections in the uncracked and cracked stage, respectively, while

$$\zeta = 1 - \alpha \left(\frac{M_{cr}}{M_{\max}} \right)^m$$

where M_{cr} is the first cracking bending moment and M_{max} is the maximum moment along the span at the load stage at which deflection is being calculated. α , β and m are factors empirically derived from test data on specific rebars and a given set up; for the most common models α, β and m values are reported in the Table 1.

Faza and GangaRao (1993) proposed a different relationship to evaluate the moment of inertia of beams; their model is based on the assumption that the concrete section of a beam between load points is fully cracked (J_2) and end sections are partially cracked (J_e). Explicit expressions of the effective moment of inertia obtained as combination of J_1 and J_2 were derived for beams with certain loading and supporting conditions.

Because of beam deflections are mainly governed by the moment-curvature law and the shape and magnitude of the moment diagram along the span, the use of empirical models cannot be generalized to the analysis of any loading and boundary conditions. Empirical models, however, are useful for design practice and are able to predict deflection values only for beams reinforced with specific FRP bars (generally Glass FRP bars) and specific loading and supporting conditions.

Table 1. α, β and *m* values

Model	α	β	***m***
ACI 440R-96	1.00	$\alpha^* (E_{FRP}/E_{steel} + 1)$	3.0
Benmokrane and oth.	0.84	1/7.00	3.0
Al-Sayed and oth.	1.00	1.00	5.5

($\alpha^*=0.5$; E_{FRP} and E_{steel} , elastic moduli of the fRP and the steel rebars)

Rational Models

These models are founded on the use of the moment-curvature law; deflection values are determined by using the principle of virtual work. The moment-curvature law can be determined in different ways referring to sections of beams (sectional models) or to blocks between two contiguous cracks (block models) (Aiello and Ombres, 2000).

In sectional models the M-χ law is determined by a numerical procedure that allows to evaluate, solving equilibrium conditions, the curvature value χ_i corresponding to the assigned bending moment M_i. Generally a layer model is adopted: the cross-section is subdivided into a number of concrete and reinforcement layers over its depth, and equilibrium conditions are imposed by supposing a constant value of strains and stresses in each layer. Such an approach allows considering the contribution of the tensile concrete supposing the perfect bond between the reinforcement and the concrete.

A simplified moment-curvature law, belonging to sectional models, was proposed from Razaqpur and oth. (2000) for calculating deflection of FRP reinforced concrete beams. In this model a trilinear moment-curvature law has been adopted considering as flexural rigidity E_cJ_g in the uncracked stage ($M<M_{cr}$) and E_cJ_{cr} in the cracked stage ($M>M_{cr}$) being J_g the moment of inertia of gross concrete section about the centroidal axis and J_{cr} the moment of inertia of cracked section transformed to concrete. Analytical relationships have been determined supposing that both the tension stiffening and the variation of the neutral axis position after cracking are negligible.

For the most common cases of loading and support conditions maximum deflection formulas were derived; the comparison with experimental data, mainly available for GFRP reinforced concrete beams, is satisfactory.

Using the block models, the moment-curvature law refers to a block between two contiguous cracks and takes into account both the contribution of the tensile concrete (tension stiffening) and the local slippage between the reinforcement and the concrete. The M-χ law is determined by a numerical procedure that allows solving a system of differential equations obtained by means of equilibrium and compatibility conditions in all blocks.

A General Method for Calculating Deflections of FRP Reinforced Concrete Beams

As evidenced, the need to correctly predict the flexural deflection of FRP reinforced concrete beams, imposes the use of a rational model that can be generalized for the analysis of any loading and boundary conditions considering the specific properties of materials and all effects connected with the interaction between the concrete and the FRP reinforcements.

At this aim, the general procedure proposed from Authors (Aiello and Ombres, 1997) seems to be essential for a reliable prediction of FRP reinforced beams deflections. The procedure, considering different cracking configurations, is derived from a cracking analysis founded on slip and bond stresses. Details about the mentioned approach are reported in previous works (Aiello and Ombres, 2000; 2001). The model can be used for all types of FRP rebars and allows to take into account, in performing structural analysis, both the slippage between the concrete and the FRP reinforcement that, in some cases, is not negligible and influences cracking and deflection values, and the tension stiffening effects that are significant in concrete members reinforced with AFRP and CFRP rebars as experimentally evidenced (Aiello, Leone and Ombres, 2001).

To develop the model it is essential to define an analytical bond-slip law between the concrete and the FRP reinforcement. Because of the wide variability of bond properties of FRP rebars available on the market, a bond-slip law generally effective isn't yet available, consequently, it has to be determined by experimental tests for each type of FRP rebars.

However, bond properties strongly influence the serviceability behavior of FRP reinforced concrete members and the serviceability conditions are critical for design purposes; it is, then, reasonable to consider bond tests, such as pullout tests and beam tests, as fundamental tests for the mechanical characterization of FRP rebars.

This problem should be considered also in Codes suggesting appropriate tests procedures for the evaluation of bond properties (maximum bond stress, bond-slip law) of FRP rebars.

Experimental Investigations

The experimental investigation was carried out on four groups of concrete beams reinforced with Carbon Fiber Reinforced Plastics (CFRP) rebars. Mechanical properties of the utilized concrete have been experimentally evaluated by standard tests in particular the mean compressive strength and the mean tensile strength was *30.53 MPa* and *2.86 MPa* respectively. CFRP rebars, with a nominal diameter of *8.26 mm*, were grain covered and helically wounded with carbon fibers to improve the bond with the concrete (Fig.1). The mean tensile strength and the mean elastic modulus of rebars, evaluated by tensile tests, were *2401 MPa* and *128799 MPa* respectively.

Figure1. FRP Rebars

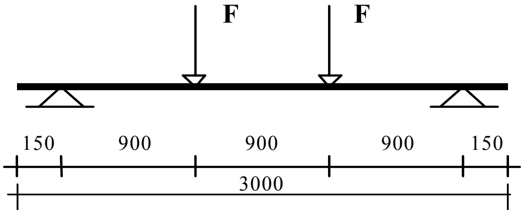


Figure 2. Scheme of the tested beam

Four groups of beams were made of *3,000 mm* length and with a rectangular cross section of *250 mm* wide and *150 mm* high while the effective depth was *130 mm*; more details about beams geometry are reported in Table 2 and sketched in the Figure 2. Two steel rebars of *8 mm* diameter have been used as reinforcement at the compression side of beams. The transversal reinforcement were realized by steel stirrups of *8 mm* diameter and *100 mm* spaced only in the zones between the applied forces and the supports in order to avoid any influences on cracks development in the constant bending zone. Near the supports and the points load steel stirrups were spaced *50 mm* and additional bended steel rebars were utilized

Details of transverse reinforcements are reported in the Figure 3.

Table 2. Details of longitudinal CFRP reinforcement of tested beams

Group	Number of Beams	Number of rebars	A_r (mm^2)
T2	2	2	107.117
T3	1	3	160.676
T6	1	6	321.352
T7	1	7	374.911

Four points flexural tests have been carried out, as shown schematically in the Figure 2. The load was applied by a hydraulic jack in load control up to failure and measured by a load cell. The displacement values were recorded by LVDT transducers at the midspan and in correspondence of load points. Strain at the compressed side of the beam were measured by electrical strain gages glued on the specimen, while deformations of tensile reinforcement were recorded by electrical strain gages glued on the CFRP rebars before casting. A displacement transducer, placed at the tensile side of the mid-span, and graduate magnifying lens were used to measure crack widths at different load levels.

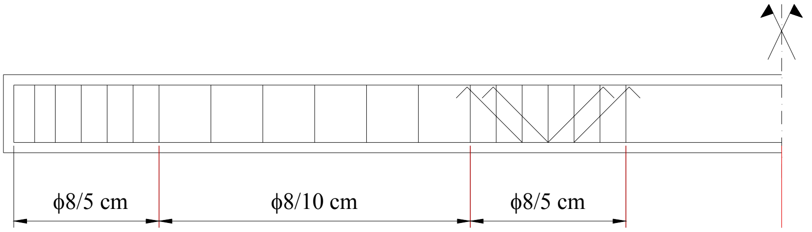


Figure 3. Details of transverse reinforcement

Bond-Slip Law

The bond-slip law was experimentally evaluated by means of a pullout test proposed by the ConFibreCrete, called Round Robin Test (Figure. 4). Utilizing the mentioned bond test bond stress, τ, and the corresponding slip, s, both at the loaded end and free end were evaluated, supposing an uniform distribution of bond stresses within the bond length:

$$\tau = \frac{F}{\pi \phi L_b}$$

where F is the applied load and L_b is the embedded length. In the Figure 5 a typical bond stress–slip curve is reported both for loaded and free end. More details about the performed experimental investigation are reported in Aiello and oth., 2001.

With reference to some theoretical models available in the literature, an analytical relationship has been calibrated able to predict the bond-slip law, on the basis of experimental obtained results. In particular, the best fit was achieved utilizing the BEP (Bertero, Popov Eligheausen) model modified as reported in Focacci and oth., 2000. Finally the following relationship has been evaluated for CFRP rebars utilized for reinforcing concrete beams

$$\tau(s) = 5.5\ s^{0.67}\left(1 - \frac{s}{24.5}\right)$$

and it has been adopted in the model for calculating deflections of beams.

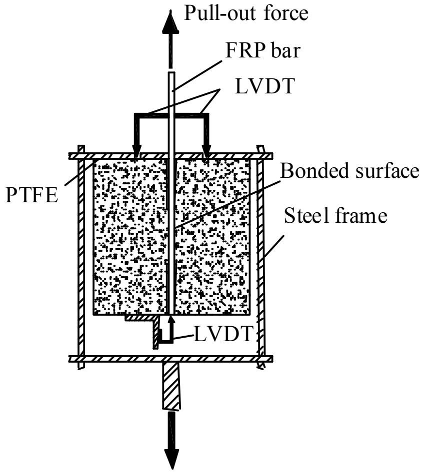


Figure 4. Pull-out test

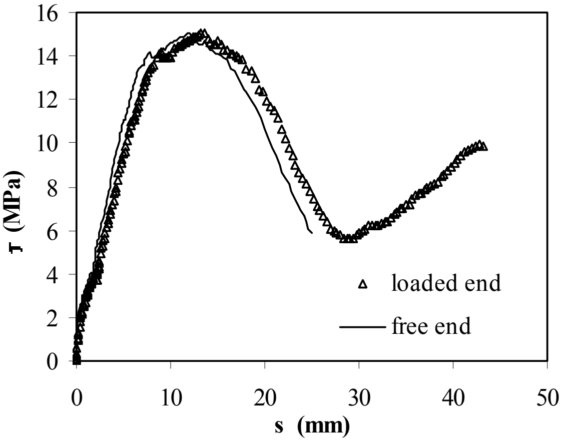


Figure 5. Bond stress- versus slips

Tension-Stiffening Evaluation

In order to analyse the contribution of the tension stiffening on cracking and deformability of FRP reinforced concrete members an experimental investigation was carried out on CFRP reinforced cylindrical concrete specimens subjected to axial tension. Specimens were made with a CFRP rebar embedded in the concrete block varying the concrete cover thickness (Figure 6). In the Figure 7 a typical F-ε curve is reported among those obtained for all tested specimens, being F the tensile applied force and ε the mean strain of the member response. In the same figure the curve F-ε, obtained for the CFRP rebars is drawn, in this case ε is the local strain in the bare bar. More details about the experimental investigation are reported in Aiello and oth., 2001.

Obtained results evidenced as the tension stiffening contribution is not negligible; in fact the rebar embedded in the concrete presents a higher stiffness, at all stress levels, with respect to the bare bar. In addition it was relieved as for a given rebar diameter it is possible to determine an optimal value of the concrete cover corresponding to a maximum tension stiffening contribution. Finally, a theoretical approach has been performed in order to predict the tension stiffening contribution; first results are presented in Aiello and oth., 2002.

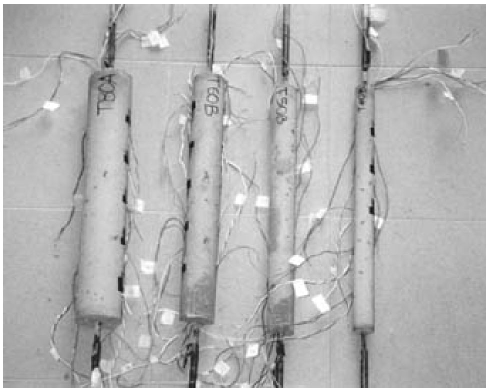


Figure 6. Tested specimens

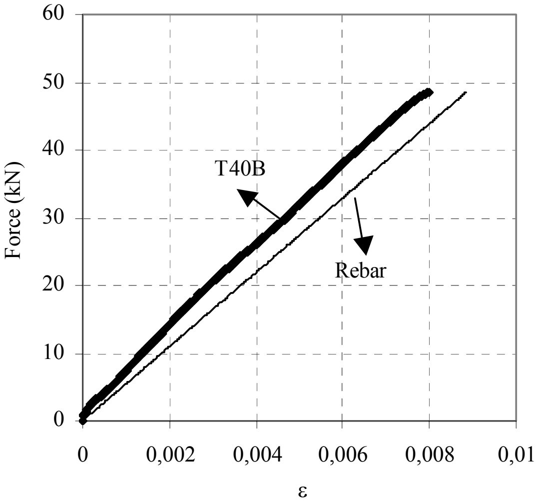
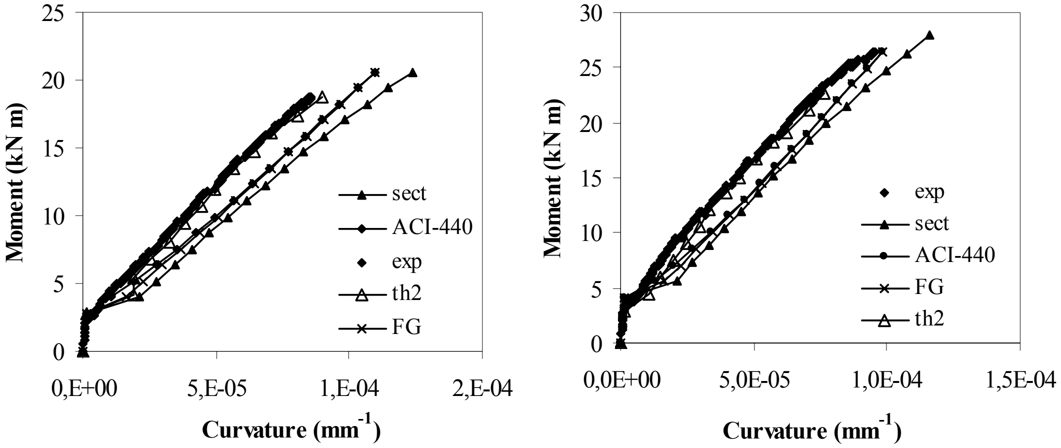


Figure 7. F-ε curves for T40B specimen

Theoretical –Experimental Comparisons

Obtained experimental results in terms of moment-curvature and load-deflections curves are reported in the following. In particular, experimental moment-curvature diagrams are compared with curves obtained by different theoretical approaches. In the Figure 8 results obtained for beams *T2B* and *T3* are drawn together with diagrams obtained utilizing the proposed model (curve *th2*), diagrams resulting from the cross-sectional approach (curve *sect*) taking into account the contribution of the tensile concrete, curves obtained by means ACI Code (ACI 440R, 1996) relationships and that proposed from Faza and GangaRao.

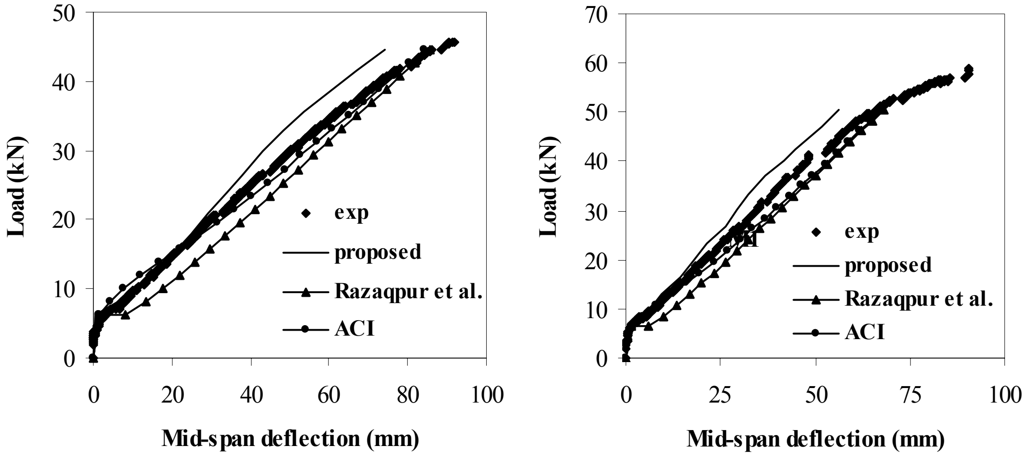


Beam T2B Beam T3

Figure 8. Moment-curvature curves: experimental and theoretical comparison

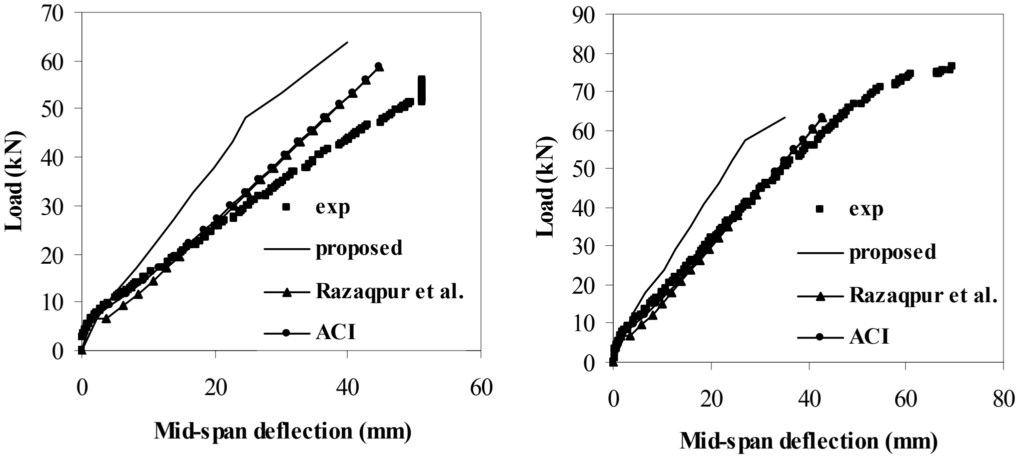
The analysis of diagrams shows as traditional theoretical approaches seem less accurately to predict the beam deformability while theoretical models, taking into account the real bond-slip laws and the tension stiffening contribution, are more appropriate, fitting very well experimental points. In all cases theoretical predictions referring to the ACI provisions and the cross-sectional model give curves with very reduced scatter.

In the Figures 9 and 10 load-deflection curves are reported for beams tested. Experimental results are compared with those obtained utilizing the block-model approach, the ACI code relationships and the theoretical approach proposed from Razaqpur and oth., 2000.



Beam T2A Beam T3

Figure 9. Midspan deflection versus applied load: theoretical and experimental comparison

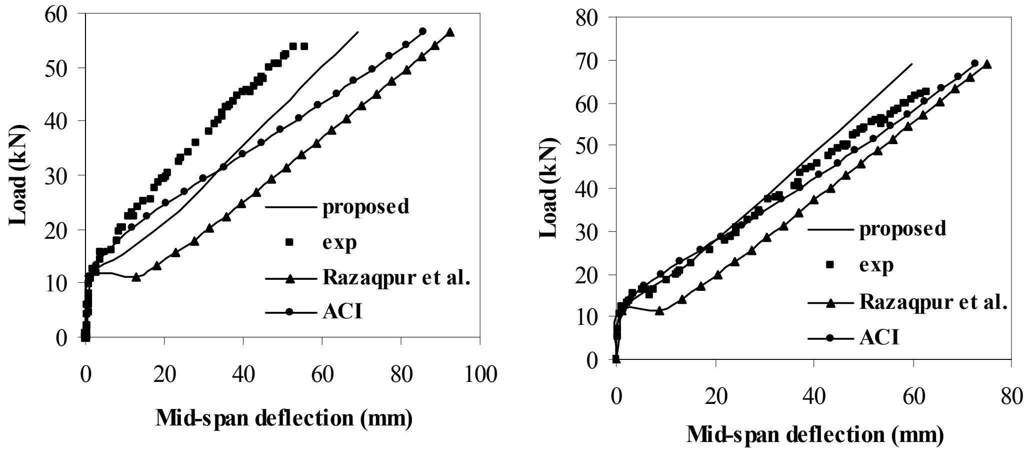


Beam T6 Beam T7

Figure 10. Midspan deflection versus applied load: theoretical and experimental results

Analysing results it is clearly evidenced as the behavior of the beams is depending on the amount of reinforcement. In fact, for lower values of reinforcement ratio (beams T2 and T3) the experimental points fall between predictions of the block-model and those obtained by other approaches while increasing the reinforcement ratio the ACI code relationship fit very well experimental points.

On the other hand, the model proposed by Razqpur gives reduced scatters with respect to experimental points only in the case of the highest reinforcement ratio (beam T7). Therefore it could be defined a reinforcement ratio value indicating the range of effectiveness of each theoretical approaches. In addition it must be emphasized as the case of low reinforcement ratio is the most critical for deformability point of view, as a consequence the use of traditional approaches, even if on the safe side, overestimate the beam deformability compromising the satisfaction of service conditions requirements.



Beam A1 (reinforcement ratio= *0.005236*) Beam A2 (reinforcement ratio= *0.005674*)

Figure 11. Midspan deflection versus applied load: comparison between theoretical and experimental results

The influence of the reinforcement ratio on the effectiveness of theoretical models was observed also in previous studies referring to AFRP reinforced concrete beams. In the Figure 11 load versus mid-span deflections are reported for two beams tested under flexure and whose detailed analysis was already presented in Aiello and Ombres, 2001. In the figure it is clear as for the beam

A1 with low reinforcement ratio, theoretical predictions overestimate deformability values, mostly referring to predictions provided by the model of Razaqpur and oth.; increasing the reinforcement ratio, for beam *A2*, the experimental points fall generally between ACI curve and the proposed block model curve. In both the analysed cases the theoretical approach proposed from Razqpur and oth. gives the most relevant scatters with regard to experimental results.

However the comparisons between results obtained for AFRP and CFRP reinforced beams allows to draw further considerations regarding the influence of the kind of reinforcement on the structural response. Increasing the reinforcement ratio, for CFRP reinforced concrete beams the classical approaches appear more appropriate than the proposed model while for AFRP reinforced concrete beams, mainly in service conditions, the proposed model gives the best fit of experimental results.

Conclusions

A general method for calculating deflections of FRP reinforced concrete was adopted in the paper. The method is found based on the use of moment-curvature diagrams of a concrete block between two contiguous cracks. The local bond-slip and the tension stiffening effects were considered.

An experimental investigation, carried out on concrete beams reinforced with Carbon FRP rebars was described and obtained results were compared with predictions of the proposed model and those of models usually adopted in the analysis of FRP reinforced concrete beams.

On the basis of obtained results, it is possible to drawn the following conclusions:

- the use of a general method in the analysis of FRP reinforced concrete beams is needed in order to take into account all parameters influencing the structural behavior of such elements;
- the wide variability of geometrical and mechanical properties of FRP rebars available on the market does not allow to define a unique model able to predict the serviceability behavior of FRP reinforced concrete beams for any load and support condition. The use of empirical models, such as Codes models, in some cases is not able for a reliable structural analysis;
- bond properties of FRP rebars and the interaction between FRP rebars and the concrete are essential for the structural analysis. Consequently, bond tests should be considered as fundamental for the mechanical characterization of FRP rebars, like tensile tests;
- the tension stiffening effects are not negligible mainly for concrete beams reinforced with high modulus FRP rebars (AFRP and CFRP rebars);
- the general method predicts very well deflections of CFRP reinforced concrete beams for low values of the reinforcement ratio, while for high values of the reinforcement ratio it underestimates experimental values;
- the proposed general method is onerous from a computational point of view and its use for design purposes is very difficult. The method, therefore, can be adopted to obtain numerical data for any load and support condition in order to define, also on the basis of the comparison with experimental results, analytical relationships (eventually simplified) useful for the design practice.

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