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Shear Strength of Concrete Beams Reinforced with FRP Bars: Design Method

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Synopsis: ACI Committee 440 has proposed a design approach for evaluating the concrete contribution to the shear resistance of FRP-reinforced concrete beams that accounts for the axial stiffness of FRP longitudinal reinforcement. Recent shear tests conducted on beams longitudinally reinforced with different types and ratios of FRP bars indicate that the current ACI 440.1R-03 shear design approach significantly underestimates the concrete shear strength of such beams. This paper presents a proposed modification to the ACI 440.1R-03 shear design equation. The proposed equation was verified against experimental shear strengths of 98 specimens tested to date, and the calculated values are shown to compare well. In addition, the proposed equation was compared to the major design provisions using the available test results. Better and consistent predictions were obtained using the proposed equation.

Keywords: axial stiffness; beams; concrete contribution; design method; FRP bars; shear strength

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INTRODUCTION

Fiber-reinforced polymer (FRP) bars are currently available as a substitute for conventional steel bars in concrete structures exposed to de-icing salts and marine environments. In addition to superior durability, FRP reinforcing bars have a high strength-to-weight ratio, which makes them attractive as reinforcement for concrete structures. However, the material properties of FRP differ significantly from those of steel reinforcement, especially the modulus of elasticity. The modulus of elasticity for commercially available glass and aramid FRP bars is 20 to 25 % that of steel compared to 60 to 75 % for carbon FRP bars.

Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced longitudinally with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This fact is supported by the findings from the experimental investigations on concrete beams without stirrups and reinforced longitudinally with carbon and glass FRP bars (El-Sayed et al. 2004 , 2005b). The investigation also revealed that the axial stiffness of the reinforcing bars is a key parameter in evaluating the concrete shear strength of flexural members reinforced with FRP bars.

The current ACI 440.1R-03 guide has proposed a design approach for calculating the concrete shear strength of FRP-reinforced concrete beams accounting for the axial stiffness of FRP reinforcing bars. Recent research has indicated that the ACI 440 shear design method provides very conservative predictions, particularly for beams reinforced with glass FRP bars (El-Sayed et al. 2004, 2005a, b, c; Razaqpur et al. 2004; Gross et al. 2004; Tureyen and Frosch 2002). Furthermore, the research has indicated that the level of conservatism of the shear strength predicted by ACI 440 method is neither consistent nor proportioned to the axial stiffness of FRP reinforcing bars (El-Sayed et al. 2005a). The objective of this paper is to present a proposed modification to the ACI 440.1R-03 shear design method. The paper also compares the predictions of the proposed equation and those by the major design provisions using the test results available in the literature.

REVIEW OF THE CURRENT DESIGN PROVISIONS

Due to the rapid increase of using FRP materials as reinforcement for concrete structures, there are international efforts to develop design guidelines. These efforts have resulted in the publishing of several codes and design guides. Most of the shear design provisions incorporated in these codes and guides are based on the design formulas of members reinforced with conventional steel considering some modifications to account for the substantial differences between FRP and steel reinforcement. These provisions use the well-known $V_c + V_s$ method of shear design, which is based on the truss analogy. This section reviews the concrete shear strength of members longitudinally reinforced with FRP bars, $V_{c,f}$, as recommended by the American Concrete Institute (ACI 440.1R-03 2003), ISIS Canada (ISIS-M03-01 2001), the Canadian Standard Association (CAN/CSA-S806-02 2002), and the Japan Society of Civil Engineers, JSCE, (Machida 1997).

ACI 440.1R-03 Design Guidelines

To account for the axial stiffness of FRP longitudinal reinforcement, A_fE_f, as compared to that of steel reinforcement, A_sE_s, ACI Committee 440 recommends the following equation for calculating $V_{c,f}$:

$$V_{c,f} = \frac{\rho_f E_f}{\rho_s E_s} V_c \tag{1}$$

where ρ_f and ρ_s are the reinforcement ratios of the flexural FRP and steel reinforcement, respectively, E_f and E_s are the modulus of elasticity of FRP and steel reinforcement, respectively, and V_c is the design shear strength provided by the concrete for the steel-reinforced section.

For practical design purposes the value of ρ_s can be taken as half the maximum reinforcement ratio allowed by ACI 318 or 0.375 ρ_{sb} and considering typical steel yield strength of 420 MPa for flexural reinforcement, the ACI Committee 440 recommends shear strength provided by concrete as follows:

$$V_{c,f} = \frac{\rho_f E_f}{90\beta_1 f_c'} \left(\frac{\sqrt{f_c'}}{6} b_w d \right) \le \frac{\sqrt{f_c'}}{6} b_w d \tag{2}$$

where $f_c^{'}$ is the specified compressive strength of concrete, β_1 is a factor defined as the ratio of the depth of equivalent rectangular stress block to the distance from the extreme compression fiber to the neutral axis, b_w is the web width, and d is the distance from extreme compression fiber to the centroid of the main tension reinforcement.

ISIS-M03-01 Design Manual

The shear resistance attributed to concrete, $V_{c,f}$, of members reinforced with FRP bars as flexural reinforcement is calculated according to the same principles as for steel reinforced concrete (CSA A23.3-94) after accounting for the difference in the modulus of elasticity between FRP and steel reinforcement as follows:

$$V_{c,f} = 0.2\lambda\phi_c\sqrt{f_c^{'}}\,b_w d\sqrt{\frac{E_f}{E_s}} \tag{3}$$

For sections with an effective depth greater than 300 mm the concrete shear resistance, $V_{c,f}$, is taken as:

$$V_{c,f} = \left[\frac{260}{1000+d}\right]\lambda\phi_c\sqrt{f_c^{'}}\,b_w d\sqrt{\frac{E_f}{E_s}} \geq 0.1\lambda\phi_c\sqrt{f_c^{'}}\,b_w d\sqrt{\frac{E_f}{E_s}} \tag{4}$$

where λ is a modification factor for density of concrete and ϕ_c is a resistance factor for concrete.

CAN/CSA-S806-02 Code

The concrete contribution to shear strength is calculated using the following equation:

$$V_{c,f} = 0.035\lambda\phi_c\left(f_c^{'}\rho_f E_f\frac{V_f}{M_f}d\right)^{1/3} b_w d \tag{5a}$$

such that:

$$0.1\lambda\phi_c\sqrt{f_c^{'}}\,b_w d \leq V_{c,f} \leq 0.2\lambda\phi_c\sqrt{f_c^{'}}\,b_w d \tag{5b}$$

$$\frac{V_f}{M_f}d \leq 1.0 \tag{5c}$$

where V_f and M_f are the factored shear force and moment at the section of interest. For sections with an effective depth greater than 300 mm and with no transverse shear reinforcement or less transverse reinforcement than the minimum required by code, the value of $V_{c,f}$ is calculated using the following equation:

$$V_{c,f} = \left(\frac{130}{1000+d}\right)\lambda\phi_c\sqrt{f_c^{'}}b_w d \geq 0.08\lambda\phi_c\sqrt{f_c^{'}}b_w d \tag{6}$$

It is evident that Eq. 6 can be derived from Eq. 4 by substituting the term $\sqrt{E_f / E_s}$ with 0.5, considering E_f = 50 GPa and E_s = 200 GPa. Thus Eq. 6 represents the lower-bound for concrete contribution to shear strength of FRP-reinforced concrete members regardless of the type of FRP reinforcing bars.

JSCE Design Recommendations

The concrete shear strength recommended by JSCE (Machida 1997) is given by the following equation:

$$V_{c,f} = \beta_d\beta_p\beta_n f_{vcd} b_w d / \gamma_b \tag{7a}$$

such that:

$$f_{vcd} = 0.2\left(f_{cd}^{'}\right)^{1/3} \leq 0.72 \text{ N/mm}^2 \tag{7b}$$

$$\beta_d = (1000/d)^{1/4} \leq 1.5 \tag{7c}$$

$$\beta_p = \left(100\rho_f E_f / E_s\right)^{1/3} \leq 1.5 \tag{7d}$$

$$\beta_n = 1 + M_o / M_d \leq 2 \qquad \text{for } N^{`}_d \geq 0$$

$$\beta_n = 1 + 2M_o / M_d \geq 0 \qquad \text{for } N^{`}_d < 0 \tag{7e}$$

where γ_b is member safety factor (γ_b =1.3), $f_{cd}^{'}$ is the design compressive strength of the concrete, M_o is the decompression moment, M_d is the design bending moment, and $N^{`}_d$ is the design axial compressive force.

PROPOSED SHEAR DESIGN EQUATION

An experimental study to investigate the shear strength of concrete beams without shear reinforcement (stirrups) and reinforced in the longitudinal direction with different types and ratios of FRP bars was carried out by the authors (El-Sayed et al. 2005b). The investigation included nine full-scale reinforced concrete beams (3250 × 250 × 400 mm) divided into three series with different reinforcement ratios (ρ = 0.87, 1.23, and 1.72%). Each series included three beams reinforced with the same reinforcement ratio of steel, carbon FRP, or glass FRP bars to explore the actual relationship between the shear strength of beams reinforced with FRP bars to that of beams reinforced with steel. The beams were tested in four-point bending over a simply supported clear span of 2750 mm, and a shear span of 1000 mm for all tests, giving a shear span-to-depth ratio of 3.1. The test results of this investigation revealed that the ratio of concrete shear strength of concrete beams reinforced with FRP bars to that of beams reinforced with steel ($V_{c,f}/V_c$) is proportional to the cube root of the axial stiffness ratio between FRP and steel reinforcing bars ($\sqrt[3]{\rho_f E_f / \rho_s E_s}$). This result explains why the current ACI 440 design method (Eq. 2) gives very conservative results, particularly for beams reinforced with glass FRP bars. The procedure followed in deriving Eq. 2 was based on the assumption that $V_{c,f}/V_c$ is directly proportioned to ($\rho_f E_f / \rho_s E_s$) (El-Sayed et al. 2005b).

Applying the above finding to Eq. 1 and following the same procedure used in deriving Eq. 2 (identified as Eq. 9-1 in ACI 440.1R-03), the following modification to Eq. 2 is proposed:

$$V_{c,f} = \left(\frac{\rho_f E_f}{90\beta_1 f_c'}\right)^{1/3} \left(\frac{\sqrt{f_c'}}{6} b_w d\right) \le \frac{\sqrt{f_c'}}{6} b_w d \tag{8a}$$

Eq. 8a can be rearranged as follows:

$$V_{c,f} = 0.037 \left(\frac{\rho_f E_f \sqrt{f_c'}}{\beta_1}\right)^{1/3} b_w d \le \frac{\sqrt{f_c'}}{6} b_w d \tag{8b}$$

According to ACI 440.1R-03, the factor β_1 in the denominator of Eq. 8b is a function of the concrete compressive strength. It can be simply expressed by the following equation:

$$0.85 \ge \beta_1 = 0.85 - 0.007(f_c' - 28) \ge 0.65 \tag{8c}$$

It is clear that Eq. 8b includes the common parameters used by civil engineers for designing reinforced concrete members.

VERIFICATION OF THE PROPOSED EQUATION

To verify the proposed modification, Eq. 8b was compared to the test results of 98 specimens tested to date as given in Table 1. These test data were collected from 15 different investigations: 3 investigations conducted by the authors and 12 investigations conducted by other researchers. The specimens included 85 beams and 13 one-way slabs; all were simply supported and were tested either in three-point or four-point bending. These specimens included 2 specimens reinforced with aramid FRP bars, 36 specimens reinforced with carbon FRP bars, and 60 specimens reinforced with glass FRP bars. All specimens had no transverse reinforcement and failed in shear. The reinforcement ratio of the test specimens ranged between 0.25 and 3.02 %. The concrete compressive strength ranged between 24.1 and 81.4 MPa; the shear span-to-depth ratio, *a/d*, ranged between 2.6 and 6.5; and the effective depth, *d*, ranged between 141 and 360 mm. Table 1 shows relevant details on the specimens included in this verification.

Besides the predicted shear strengths according to the proposed equation (Eq. 8b), the predicted shear capacities according to the current ACI 440 shear design equation (Eq. 2) are also presented in Table 1. For the 98 tests, the average V_{exp}/V_{pred} for the proposed equation is 1.31 with a coefficient of variation of 17.5 %. On the other hand, these averages were 3.55 and 38.3 % for the current ACI 440 method. Figure 1 shows a comparison between the experimental and predicted shear strengths based on the results of the proposed and current equations. The vertical axis in this figure represents the ratio V_{exp}/V_{pred}, while the horizontal axis represents the axial stiffness ($\rho_f E_f$) of FRP reinforcing bars. From Fig. 1 and Table 1, it is evident that the level of accuracy of the shear strength predicted by the proposed equation is consistent with the varying reinforcement ratio (ρ_f) and type (E_f) of FRP reinforcing bars unlike the current method of ACI 440. The same observation can be made when the results of the proposed equation and those of the current ACI 440 equation are plotted versus the concrete strength, *a/d* ratio, and effective depth as in Figs. 2, 3, and 4, respectively. Across the range of variables included in the data, the predictions of the current ACI design method appear to have larger band width of the scattered results and higher level of conservatism compared to that of the proposed equation. Thus, the proposed equation (Eq. 8b) appears to be more accurate and reliable for predicting the concrete shear strength for flexural members longitudinally reinforced with FRP bars.

COMPARISON WITH MAJOR DESIGN PROVISIONS

To further verify the proposed equation, the predictions from Eq. (8b) were also compared with the predictions from the major design provisions. The comparison was made using the same 98 test data from the literature. In addition to the predictions of the proposed equation and those of the ACI 440.1R-03, Table 1 also gives the predictions of ISIS-M03-01 (2001) design manual, CAN/CSA-S806-02 (2002) code and JSCE design recommendations (Machida 1997). Also, the design equation recently developed by Tureyen and Frosch (2003) was used in the comparison. This equation was developed from a model that calculates the concrete contribution to shear strength of reinforced concrete beams. The equation was simplified to provide a design formula applicable to both steel and FRP-reinforced beams as follows:

$$V_c = \frac{2}{5}\sqrt{f_c'}b_w c \qquad (9)$$

where $c = kd$ = cracked transformed section neutral axis depth, mm

$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$

n_f = ratio of the modulus of elasticity of FRP bars to the modulus of elasticity of concrete

For predictions using ISIS manual and CSA code, the applicability conditions regarding the depth of the member was taken into account. For members with an effective depth less than 300 mm, Eqs. 3 and 5 were considered for ISIS and CSA predictions, respectively. While for members with an effective depth greater than 300 mm, Eqs. 4 and 6 were considered for the predictions by the two methods, respectively. In addition, the factors λ and ϕ_c in those equations were taken equal to 1 and the values of V_f and M_f in Eq.5 were calculated in the shear span at a distance d away from the point load. Equation 7 was used for predicting the shear strengths according to the JSCE method. It should be pointed out that the member safety factor γ_b was taken equal to 1.0 and the design axial compressive force $N`_d$ as well as the decompression moment M_o was taken equal to zero.

It can be noticed from Table 1 that although the mean value of V_{exp}/V_{pred} obtained by ISIS manual and CSA code are approximately the same as that from the proposed equation (V_{exp}/V_{pred} = 1.27 and 1.31 for ISIS and CSA, respectively), the predictions from both provisions have more scatter and are less consistent as the coefficient of variation for the two methods are 29.9 and 29 %, respectively, compared to 17.5 % for the proposed equation. On the other hand, Eq. 9 developed by Tureyen and Frosch (2003) gives consistent predictions similar to the proposed equation as the coefficient of variation for this method is 17.6 %. However, Eq. 9 gives more conservative predictions since the mean value of V_{exp}/V_{pred} by this equation is 1.87 compared to 1.31 by the proposed equation. Comparable results to those obtained by the proposed equation can be attained by the JSCE method as the mean value of V_{exp}/V_{pred} and the coefficient of variation by this method are 1.32 and 19.7%, respectively. Nevertheless, the proposed equation has the advantage of being much simpler than the JSCE equation, which requires more calculations.

CONCLUSIONS

A proposed modification to the current shear design equation in ACI 440.1R-03 (Eq. 9-1) is presented. This modification is based on experimental findings which represent the potential of empirical and semi-empirical formulations. The proposed equation was used to calculate the shear strength of 98 specimens tested in 15 different investigations. It was found that the proposed equation gives accurate predictions and yet conservative over the range of variables known to affect the concrete shear strength. To further verify this modification, the proposed equation was compared to the major design provisions using the available test data. More accurate and consistent predictions were obtained using the proposed equation.

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Table 1 — Verification of the proposed modification and comparison with major design provisions.

Reference	beam	f_c` (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)	V_{exp}/V_{pred}					
						ρ_f (%)	E_f (GPa)		Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
El-Sayed et al. (2005a)	S-C1	40.0	1000	165.3	1000.0	0.39	114.0	140.0	1.48	5.08	0.89	1.59	1.36	2.12
	S-C2B	40.0	1000	165.3	1000.0	0.78	114.0	167.0	1.40	3.01	1.06	1.51	1.29	1.85
	S-C3B	40.0	1000	160.5	1000.0	1.18	114.0	190.0	1.43	2.29	1.24	1.56	1.32	1.82
	S-G1	40.0	1000	162.1	1000.0	0.86	40.0	113.0	1.32	5.36	1.23	1.44	1.22	1.96
	S-G2	40.0	1000	159.0	1000.0	1.70	40.0	142.0	1.35	3.51	1.58	1.48	1.25	1.85
	S-G2B	40.0	1000	162.1	1000.0	1.71	40.0	163.0	1.52	3.87	1.78	1.65	1.40	2.07
	S-G3	40.0	1000	159.0	1000.0	2.44	40.0	163.0	1.37	2.69	1.81	1.50	1.27	1.81
	S-G3B	40.0	1000	154.1	1000.0	2.63	40.0	168.0	1.42	2.66	1.93	1.58	1.32	1.86
El-Sayed et al. (2005b)	CN-1	50.0	250	326.0	1000.0	0.87	128.0	77.5	1.14	2.25	0.86	1.37	1.21	1.48
	GN-1	50.0	250	326.0	1000.0	0.87	39.0	70.5	1.54	6.71	1.41	1.25	1.64	2.31
	CN-2	44.6	250	326.0	1000.0	1.24	134.0	104.0	1.39	2.01	1.19	1.95	1.45	1.73
	GN-2	44.6	250	326.0	1000.0	1.22	42.0	60.0	1.18	3.80	1.23	1.12	1.23	1.68
	CN-3	43.6	250	326.0	1000.0	1.72	134.0	124.5	1.50	1.75	1.44	2.36	1.56	1.83
	GN-3	43.6	250	326.0	1000.0	1.71	42.0	77.5	1.38	3.51	1.60	1.47	1.44	1.87
El-Sayed et al. (2005c)	CH-1.7	63.0	250	326.0	1000.0	1.71	135.0	130.0	1.41	1.94	1.25	2.05	1.60	1.71
	GH-1.7	63.0	250	326.0	1000.0	1.71	42.0	87.0	1.39	4.16	1.50	1.37	1.58	1.90
	CH-2.2	63.0	250	326.0	1000.0	2.20	135.0	174.0	1.74	2.02	1.67	2.74	1.97	2.07
	GH-2.2	63.0	250	326.0	1000.0	2.20	42.0	115.5	1.70	4.30	1.99	1.82	1.93	2.26

Table 1 (cont.) — Verification of the proposed modification and comparison with major design provisions.

Reference	beam	f_c' (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)	V_{exp}/V_{pred}					
						ρ_f (%)	E_f (GPa)		Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
Razaqpur et al. (2004)	BR1	40.5	200	225.0	600.0	0.25	145.0	36.1	1.49	5.79	0.74	1.11	1.42	2.20
	BR2	49.0	200	225.0	600.0	0.50	145.0	47.0	1.45	3.81	0.88	1.08	1.40	1.98
	BR3	40.5	200	225.0	600.0	0.63	145.0	47.2	1.43	3.00	0.97	1.07	1.37	1.90
	BR4	40.5	200	225.0	600.0	0.88	145.0	42.7	1.16	1.94	0.88	0.86	1.11	1.48
	BA3	40.5	200	225.0	800.0	0.50	145.0	49.7	1.63	3.98	1.02	1.40	1.55	2.21
	BA4	40.5	200	225.0	950.0	0.50	145.0	38.5	1.26	3.08	0.79	1.17	1.21	1.71
Gross et al. (2004)	8-2a	60.3	127	143.0	910.0	0.33	139.0	14.3	1.20	4.69	0.61	1.30	1.19	2.04
	8-2b	60.3	127	143.0	910.0	0.33	139.0	12.9	1.08	4.21	0.55	1.17	1.07	1.83
	8-2c	60.3	127	143.0	910.0	0.33	139.0	14.7	1.24	4.81	0.63	1.34	1.23	2.10
	8-3a	61.8	159	141.0	910.0	0.58	139.0	19.8	1.11	3.03	0.68	1.21	1.11	1.77
	8-3b	61.8	159	141.0	910.0	0.58	139.0	23.1	1.30	3.53	0.79	1.41	1.29	2.06
	8-3c	61.8	159	141.0	910.0	0.58	139.0	17.0	0.96	2.60	0.58	1.04	0.95	1.52
	11-2a	81.4	89	143.0	910.0	0.47	139.0	8.8	0.89	3.34	0.46	0.92	0.93	1.33
	11-2b	81.4	89	143.0	910.0	0.47	139.0	11.7	1.19	4.46	0.61	1.22	1.24	1.78
	11-2c	81.4	89	143.0	910.0	0.47	139.0	8.9	0.90	3.40	0.47	0.93	0.94	1.36
	11-3a	81.4	121	141.0	910.0	0.76	139.0	14.3	0.92	2.52	0.56	0.96	0.96	1.30
	11-3b	81.4	121	141.0	910.0	0.76	139.0	15.3	0.99	2.69	0.60	1.02	1.03	1.39
	11-3c	81.4	121	141.0	910.0	0.76	139.0	16.6	1.07	2.91	0.64	1.11	1.11	1.51

Table 1 (cont.) — Verification of the proposed modification and comparison with major design provisions.

Reference	beam	f_c' (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement ρ_f (%)	Reinforcement E_f (GPa)	V_{exp} (kN)	V_{exp}/V_{pred} Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
Tariq and Newhook (2003)	G07N1	37.3	160	346.0	951.5	0.72	42.0	54.5	1.99	8.41	1.82	1.67	2.12	3.00
	G07N2	37.3	160	346.0	951.5	0.72	42.0	63.7	2.32	9.83	2.13	1.95	2.48	3.51
	G10N1	43.2	160	346.0	1149.0	1.10	42.0	42.7	1.30	4.39	1.33	1.22	1.37	1.86
	G10N2	43.2	160	346.0	1149.0	1.10	42.0	45.5	1.38	4.68	1.41	1.29	1.46	1.98
	G15N1	34.1	160	325.0	1150.5	1.54	42.0	48.7	1.51	3.68	1.78	1.63	1.59	2.07
	G15N2	34.1	160	325.0	1150.5	1.54	42.0	44.9	1.39	3.39	1.64	1.51	1.46	1.91
	C07N1	37.3	130	310.0	949.0	0.72	120.0	49.2	1.74	3.65	1.30	2.01	1.80	2.31
	C07N2	37.3	130	310.0	949.0	0.72	120.0	45.8	1.62	3.40	1.21	1.88	1.68	2.15
	C10N1	43.2	130	310.0	1150.0	1.10	120.0	47.6	1.40	2.35	1.17	1.81	1.44	1.79
	C10N2	43.2	130	310.0	1150.0	1.10	120.0	52.7	1.55	2.61	1.29	2.00	1.59	1.98
	C15N1	34.1	130	310.0	1150.0	1.54	120.0	55.9	1.57	1.91	1.55	2.39	1.64	1.95
	C15N2	34.1	130	310.0	1150.0	1.54	120.0	58.3	1.64	1.99	1.61	2.50	1.71	2.03
Gross et al. (2003)	1a-26	79.6	203	225.0	914.0	1.25	40.3	41.6	1.29	5.66	1.14	1.10	1.38	1.86
	1b-26	79.6	203	225.0	914.0	1.25	40.3	30.4	0.94	4.14	0.83	0.81	1.01	1.36
	1c-26	79.6	203	225.0	914.0	1.25	40.3	42.1	1.30	5.72	1.15	1.12	1.39	1.88
	2a-26	79.6	152	225.0	914.0	1.66	40.3	31.0	1.16	4.24	1.13	1.00	1.25	1.62
	2b-26	79.6	152	225.0	914.0	1.66	40.3	33.1	1.24	4.52	1.21	1.07	1.33	1.73
	2c-26	79.6	152	225.0	914.0	1.66	40.3	33.5	1.26	4.58	1.22	1.08	1.35	1.75
	3a-27	79.6	165	224.0	914.0	2.10	40.3	38.4	1.23	3.85	1.30	1.06	1.32	1.67

Table 1 (cont.) — Verification of the proposed modification and comparison with major design provisions.

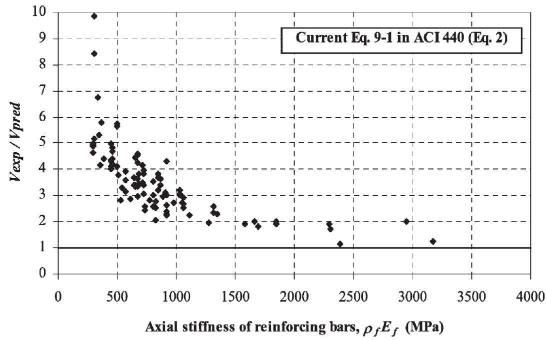
Reference	beam	f_c' (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)	V_{exp}/V_{pred}					
						ρ_f (%)	E_f (GPa)		Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
Gross et al. (2003) (cont.)	3b-27	79.6	165	224.0	914.0	2.10	40.3	32.2	1.03	3.22	1.09	0.89	1.11	1.40
	3c-27	79.6	165	224.0	914.0	2.10	40.3	36.7	1.18	3.67	1.24	1.01	1.26	1.60
	4a-37	79.6	203	224.0	914.0	2.56	40.3	48.3	1.18	3.22	1.33	1.06	1.26	1.56
	4b-37	79.6	203	224.0	914.0	2.56	40.3	45.7	1.12	3.05	1.26	0.96	1.20	1.48
	4c-37	79.6	203	224.0	914.0	2.56	40.3	45.2	1.10	3.02	1.24	0.95	1.18	1.46
Tureyen and Frosch (2002)	V-G1-1	39.7	457	360.0	1219.2	0.96	40.5	108.1	1.20	4.41	1.21	1.09	1.29	1.75
	V-G2-1	39.9	457	360.0	1219.2	0.96	37.6	94.7	1.08	4.16	1.10	0.95	1.15	1.59
	V-A-1	40.3	457	360.0	1219.2	0.96	47.1	114.8	1.21	4.03	1.18	1.15	1.29	1.73
	V-G1-2	42.3	457	360.0	1219.2	1.92	40.5	137.0	1.18	2.81	1.49	1.34	1.27	1.60
	V-G2-2	42.5	457	360.0	1219.2	1.92	37.6	152.6	1.35	3.38	1.72	1.49	1.45	1.83
	V-A-2	42.6	457	360.0	1219.2	1.92	47.1	177.0	1.45	3.13	1.78	1.72	1.55	1.92
Yost et al. (2001)	1FRPa	36.3	229	225.0	914.0	1.11	40.3	39.1	1.36	4.36	1.40	1.24	1.30	2.27
	1FRPb	36.3	229	225.0	914.0	1.11	40.3	38.5	1.33	4.29	1.38	1.22	1.28	2.23
	1FRPc	36.3	229	225.0	914.0	1.11	40.3	36.8	1.28	4.10	1.32	1.17	1.22	2.13
	2FRPa	36.3	178	225.0	914.0	1.42	40.3	28.1	1.15	3.15	1.30	1.06	1.11	1.87
	2FRPb	36.3	178	225.0	914.0	1.42	40.3	35.0	1.44	3.93	1.61	1.32	1.38	2.33
	2FRPc	36.3	178	225.0	914.0	1.42	40.3	32.1	1.32	3.60	1.48	1.21	1.26	2.14
	3FRPa	36.3	229	225.0	914.0	1.66	40.3	40.0	1.21	2.98	1.43	1.11	1.16	1.93
	3FRPb	36.3	229	225.0	914.0	1.66	40.3	48.6	1.47	3.62	1.74	1.35	1.41	2.34

Table 1 (cont.) — Verification of the proposed modification and comparison with major design provisions.

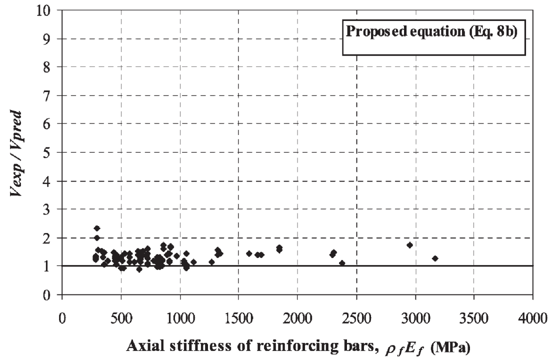
Reference	beam	f_c` (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement ρ_f (%)	Reinforcement E_f (GPa)	V_{exp} (kN)	V_{exp}/V_{pred} Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
Yost et al. (2001) (cont.)	3FRPc	36.3	229	225.0	914.0	1.66	40.3	44.7	1.36	3.33	1.60	1.24	1.30	2.15
	4FRPa	36.3	279	225.0	914.0	1.81	40.3	43.8	1.06	2.46	1.30	0.97	1.02	1.67
	4FRPb	36.3	279	225.0	914.0	1.81	40.3	45.9	1.11	2.58	1.35	1.02	1.06	1.74
	4FRPc	36.3	279	225.0	914.0	1.81	40.3	46.1	1.11	2.59	1.36	1.02	1.07	1.75
	5FRPa	36.3	254	224.0	914.0	2.05	40.3	37.7	0.96	2.06	1.22	0.89	0.92	1.49
	5FRPb	36.3	254	224.0	914.0	2.05	40.3	51.0	1.30	2.79	1.66	1.20	1.25	2.02
	5FRPc	36.3	254	224.0	914.0	2.05	40.3	46.6	1.19	2.55	1.51	1.10	1.14	1.85
	6FRPa	36.3	229	224.0	914.0	2.27	40.3	43.5	1.19	2.38	1.57	1.10	1.14	1.83
	6FRPb	36.3	229	224.0	914.0	2.27	40.3	41.8	1.15	2.29	1.51	1.05	1.10	1.76
	6FRPc	36.3	229	224.0	914.0	2.27	40.3	41.3	1.13	2.26	1.49	1.04	1.09	1.74
Alkhrdaji et al. (2001)	BM7	24.1	178	279.0	750.0	2.30	40.0	53.4	1.66	2.63	2.45	1.30	1.75	2.24
	BM8	24.1	178	287.0	750.0	0.77	40.0	36.1	1.57	5.17	1.61	1.21	1.67	2.40
	BM9	24.1	178	287.0	750.0	1.34	40.0	40.1	1.45	3.30	1.79	1.12	1.54	2.07
Deitz et al. (1999)	GFRP1	28.6	305	157.5	710.0	0.73	40.0	26.8	1.22	4.66	1.17	1.19	1.15	1.85
	GFRP2	30.1	305	157.5	913.0	0. 73	40.0	28.3	1.27	4.99	1.20	1.38	1.20	1.93
	GFRP3	27.0	305	157.5	913.0	0. 73	40.0	29.2	1.35	4.96	1.31	1.47	1.28	2.05
	Hybrid1	28.2	305	157.5	913.0	0. 73	40.0	28.5	1.31	4.95	1.25	1.42	1.24	1.98
	Hybrid2	30.8	305	157.5	913.0	0. 73	40.0	27.6	1.24	4.89	1.16	1.33	1.16	1.87

Table 1 (cont.) — Verification of the proposed modification and comparison with major design provisions.

Reference	beam	f_c` (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)	V_{exp}/V_{pred}					
						ρ_f (%)	E_f (GPa)		Proposed Equation (Eq. 8b)	Current ACI 440 Eq. (Eq. 2)	ISIS M03-01 (Eqs. 3 & 4)	CSA –S806-02 (Eqs. 5 & 6)	JSCE (Eq. 7)	Tureyen and Frosch (Eq. 9)
Mizukawa et al. (1997)	No.1	34.7	200	260.0	700.0	1.30	130.0	62.2	1.39	1.81	1.26	1.05	1.38	1.74
Duranovic et al. (1997)	GB6	32.9	150	210.0	766.5	1.36	45.0	22.0	1.15	2.87	1.28	1.01	1.09	1.59
Swamy and Aburawi (1997)	F-6-GF	39.0	154	222.0	700.0	1.55	34.0	19.50	0.95	2.82	1.11	0.77	0.90	1.33
Zhao et al. (1995)	No.1	34.3	150	250.0	750.0	1.51	105.0	45.0	1.43	1.93	1.41	1.14	1.41	1.79
	No.6	34.3	150	250.0	750.0	3.02	105.0	46.0	1.26	1.25	1.45	0.93	1.14	1.39
	No.15	34.3	150	250.0	750.0	2.27	105.0	40.5	1.12	1.15	1.27	0.90	1.11	1.37
Mean									**1.31**	**3.55**	**1.27**	**1.31**	**1.32**	**1.87**
Standard deviation									**0.23**	**1.36**	**0.38**	**0.38**	**0.26**	**0.33**
Coefficient of variation (%)									**17.5**	**38.3**	**29.9**	**29.0**	**19.7**	**17.6**



(a)



(b)

Fig. 1— Experimental-to-predicted shear strength versus axial stiffness of reinforcing bars: (a) Current equation in ACI 440; (b) Proposed equation.

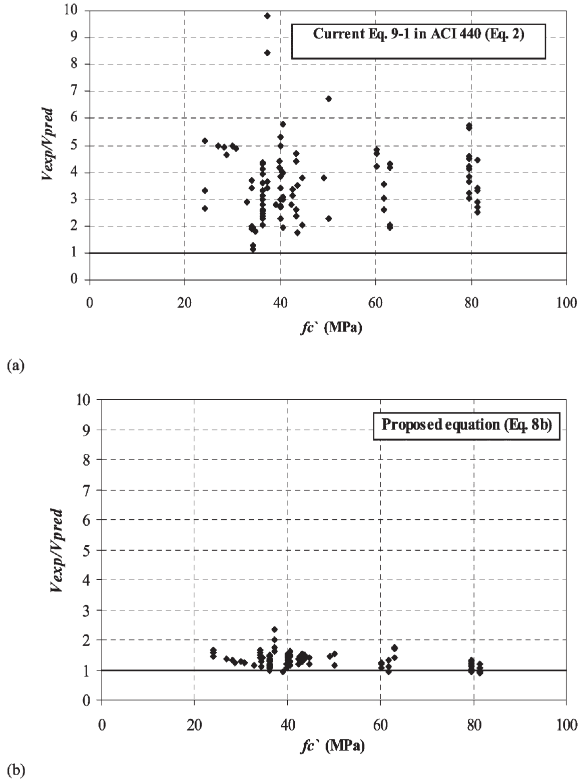
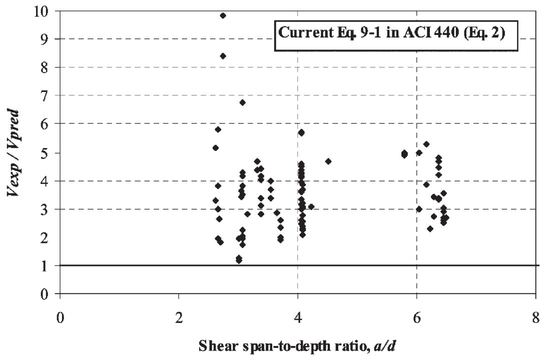
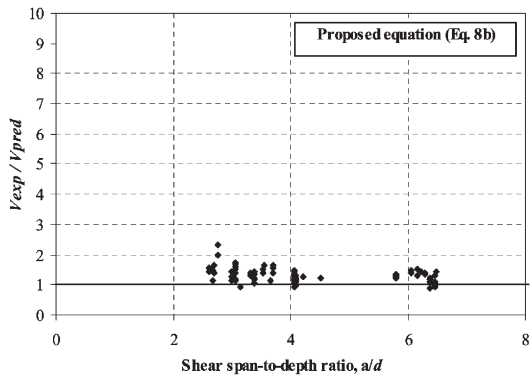


Fig. 2— Experimental-to-predicted shear strength versus concrete compressive strength: (a) Current equation in ACI 440; (b) Proposed equation.



(a)



(b)

Fig. 3— Experimental-to-predicted shear strength versus shear span-to-depth ratio: (a) Current equation in ACI 440; (b) Proposed equation.

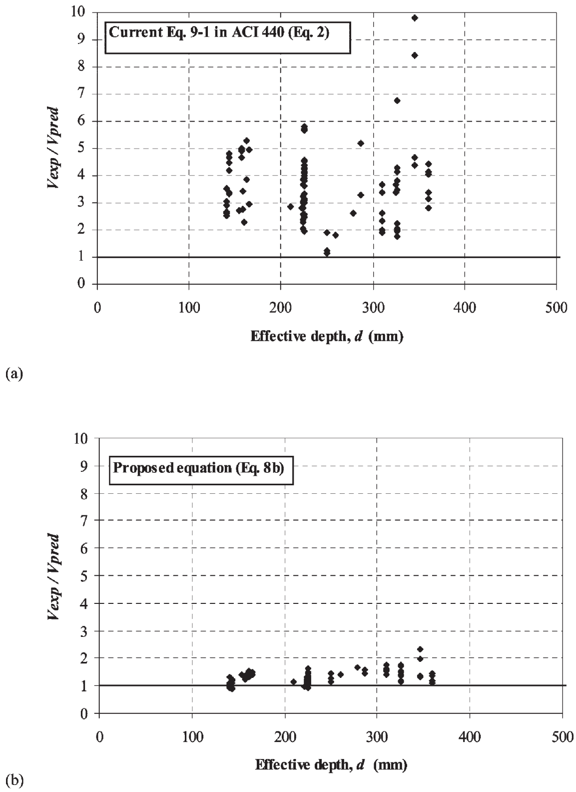


Fig. 4— Experimental-to-predicted shear strength versus effective depth: (a) Current equation in ACI 440; (b) Proposed equation.