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Nonlinear constitutive models for FRP composites using artificial neural networks

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Abstract

This paper presents a new approach to generate nonlinear and multi-axial constitutive models for fiber reinforced polymeric (FRP) composites using artificial neural networks (ANNs). The new nonlinear ANN constitutive models are complete and have been integrated with displacement-based FE software for the nonlinear analysis of composite structures. The proposed ANN constitutive models are trained with experimental data obtained from off-axis tension/compression and pure shear (Arcan) tests. The proposed ANN constitutive model is generated for plane–stress states with assumed functional response in some parts of the multi-axial stress space with no experimental data. The ability of the trained ANN models to predict material response is examined directly and through FE analysis of a notched composite plate. The experimental part of this study involved coupon testing of thick-section pultruded FRP E-glass/polyester material. Nonlinear response was pronounced including in the fiber direction due to the relatively low overall fiber volume fraction (FVF). Notched composite plates were also tested to verify the FE, with ANN material models, to predict general non-homogeneous responses at the structural level.

Keywords: Artificial neural network (ANN); Fiber reinforced polymer (FRP) composite; Nonlinear constitutive models; Finite element analysis (FEA); Plane stress–strain behavior

1. Introduction

Thick-section FRP composites, such as those manufactured by the pultrusion process, usually have a relatively low fiber volume fraction (FVF) with relatively thick-sections (e.g. from 0.3175 cm to 2.54 cm). Thick-section composites can be composed of multi-layers using unidirectional roving and continuous filament mat (CFM) layers that are repeated through the thickness direction. They exhibit nonlinear stress–strain behaviors due to the low FVF and large thickness. In addition, nonlinear response is magnified due to manufacturing anomalies (e.g. inclusion, void and micro-crack). Haj-Ali and Kilic (2002), Kilic and Haj-Ali (2003) investigated the nonlinear behavior of thick-section and multi-layered FRP composites and proposed nonlinear macro- and micro-mechanical models. Their models were able to reasonably capture the multi-axial response. El-Hajjar and Haj-Ali (2004)

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proposed a testing method to measure the in-plane shear response of FRP composites under multi-axial deformation using a modified Arcan test fixture. Haj-Ali and Muliana (2003, 2004), Muliana and Haj-Ali (2006) have proposed constitutive models to generate the nonlinear mechanical and time-dependent behavior of the FRP composite.

In order to approximate the complex material or structural behaviors in composites, artificial neural networks have been proposed as an alternative modeling approach for both material and structural behaviors due to their effectiveness, robustness, and noise-tolerance. Artificial neural networks synthesized with a number of training data can effectively predict the nonlinear multi-axial stress–strain relations by capturing and generalizing complex behaviors in their connection weights among artificial neurons, even though they are not easily approximated by conventional methods, e.g. Wasserman (1989).

Pidaparti and Palakal (1993) were among the first to use ANN as a constitutive model for composites. Uni-axial and off-axis tests from angle-ply laminates were used. Artificial neural networks were trained to include off-axis angle, initial stress, and incremental stress as input variables. They demonstrated that their models can predict uni-axial stress–strain behaviors with different off-axis angle and verified them with experiments. However, these ANN models were limited because they cannot provide full multi-axial plane–stress constitutive models, required to perform structural analysis of laminated composites. Therefore, there is a need for a full and complete nonlinear ANN material model that can cover the entire nonlinear stress–strain spectrum, including the tensile-compression-shear stress paths. Okafor et al. (1996) used ANN to predict the delamination length in laminated composite beams. Their ANN models were trained with simulation results (i.e. normalized natural frequencies of damaged composite beams as a function of the induced delamination length for first four modes). They found that a learning rate of 0.3 is proper for their ANNs and compared the predicted delamination length with corresponding test data. Chakraborty (2005) proposed an ANN delamination model in order to predict the shape, size, and location of delaminations in laminated specimens with an elliptical embedded delamination. Finite element simulations were used to generate the ANN training data with different delamination geometry and location as output variables. The natural frequencies for up to 10 first modes were used as input variables. The trained ANN model was able to predict FE results for delamination cases that were not used in the training process. This ANN modeling approach can be used for future NDE damage detection in structural composites. Ghaboussi et al. (1998) suggested an effective ANN training method, which is termed Autoprogressive Training in order to effectively train ANNs when a small number of training data (e.g. experimental responses measured from structural tests) are available. They used experimental results performed on laminated graphite/epoxy plates with a hole to train a neural network material model using the Autoprogressive training method and verify the trained ANN model by comparing to other experimental results. Ootao et al. (1999) applied a neural network approach to optimize the material composition of functionally graded material with respect to the thermal stress distribution in a hollow circular cylinder. Haj-Ali et al. (2001) used ANNs to generate nonlinear micro-mechanical models for unidirectional fiber reinforced materials including damage behavior in the form of interfacial cracks between the fiber and matrix. The crack angle was used as a damage parameter in their unit cell (UC) models. They demonstrated the effectiveness of their ANN models by comparing with FE results that were used in the training process. Zhang et al. (2002) investigated the correlation between temperature and dynamic mechanical properties (i.e. storage modulus and damping) for short carbon fiber composites with two polymeric matrices, PTFE and PEEK. They used ANN to generate those relations and showed good approximation when compared with experimental results.

Artificial neural networks have also been used in "inverse problems". In this mode, the trained ANN is not only used to approximate the response, but also applied directly/indirectly to generate the best parameters that minimize the error between its input/output spectrums against a given measured data. Muliana et al. (2002) proposed ANNs to generate models for the monotonic part of the nanoindentation response (i.e. load-deflection curves) of a substrate or film. They showed the potential of using their trained ANN models for a wide range of nonlinear materials. Their ANN models can be used to extract the nonlinear stress–strain parameters of the film or substrate from a given nanoindentation response. Similar study has been performed by Huber et al. (2002). The latter also used ANNs to generate material parameters of the film and substrate from indirect nanoindentation responses, and relied on both monotonic and unloading portions of the indentation curve.

The goal of this study is to develop new ANN constitutive models, which can generate nonlinear multi-axial stress–strain behaviors of FRP composites for the entire plane-stress constitutive domain. Toward this goal, different structural ANN constitutive models are generated and trained with experimental data obtained from off-axis tension/compression tests and pure shear tests. Their predictability and efficiency are examined by comparing experimental data in the local and global material directions. In addition, the proposed ANN is synthesized with FEA software as a user-defined material module for FRP composites. The FE software, integrated with trained ANN constitutive models, is used to simulate the response of a notched composite plate. Structural responses (e.g. stress–strain behavior), from the FE simulation, are compared with experimental results in order to demonstrate the ability of the ANN constitutive model to generate full-span multi-axial behavior and that it can be coupled with standard FEA software as user-defined nonlinear material models.

2. Proposed ANN nonlinear constitutive models

The proposed ANN material models are illustrated in Fig. 1. It describes a state of plane–stress for a layer or effective state in multiple orthotropic layers. The objective of the trained ANN is to generate multi-axial stress–strain relations. This can be achieved in several ways by using different ANN structures, e.g. type of input and outputs, incremental, or total variables. In this study, a general four-layer feed-forward ANN structure is used. The four-layers consist of one input, two feed-forward hidden

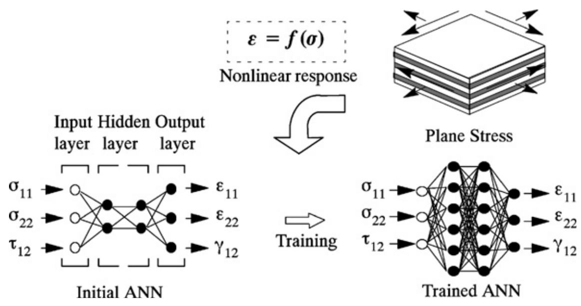

Fig. 1. Schematic drawing of a typical feed-forward 4-layer ANN structure for plane–stress nonlinear material models.

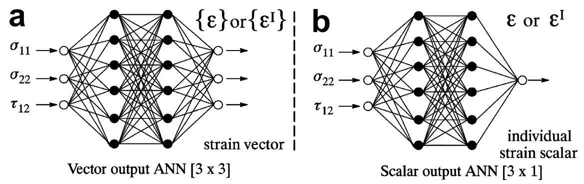

Fig. 2. Schematic drawing of two ANN general models using total and/or inelastic strain. The second approach in (b) is to generate ANNs for each individual output variable.

layers, and one output layer. Fig. 2 shows a schematic drawing of different ANN structures used in this study. The training process is carried out using a relatively large set of input and output data. The ANN structure is initially composed of a small number of neurons in the two hidden layers. The training algorithm developed in this study allows adding neurons in the hidden layers at specified intervals during the training. Fig. 1 illustrates the adaptive nature during the ANN training. The developed software relies on the conjugate gradient method to minimize the total error and find the internal connection weights, e.g. Wasserman (1989). Four different combination of ANN models are used in this study as shown in Table 2. The total error definition is expressed as:

$$\text{error} = \frac{1}{2}\sum_{n=1}\sum_{i}\|T_{\mathrm{i}} - O_{\mathrm{i}}\|^2 \tag{1}$$

where T and O are input and output vectors.

Four types of ANNs were generated with different outputs. All four ANNs have the same input: three stress components. These ANNs are classified based on their scalar or vector output and using total or inelastic strains.

3. Coupon tests for stress–strain training data

Training data for the proposed ANNs were collected from off-axis compression and tension tests performed with coupons cut from a monolithic composite plate manufactured by pultrusion process as shown in Fig. 3. Haj-Ali and Kilic (2002) used similar tests to calibrate micromodels for this material. In addition to the off-axis tests, limited axial-shear stress–strain relations were generated using a modified Arcan fixture developed by El-Hajjar and Haj-Ali (2004). All in-plane stress–strain data sets were collected up to ultimate failure state using rosette strain gages (0°, 45° and 90° strain directions) installed at the center of each coupon and

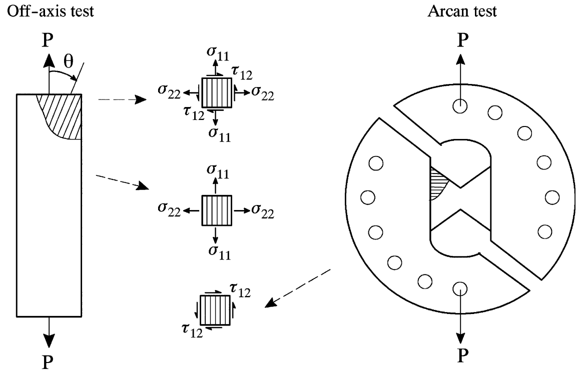

Fig. 3. Off-axis and Arcan bi-axial coupon tests performed to generate multi-axial stress–strain data needed for the ANN training.

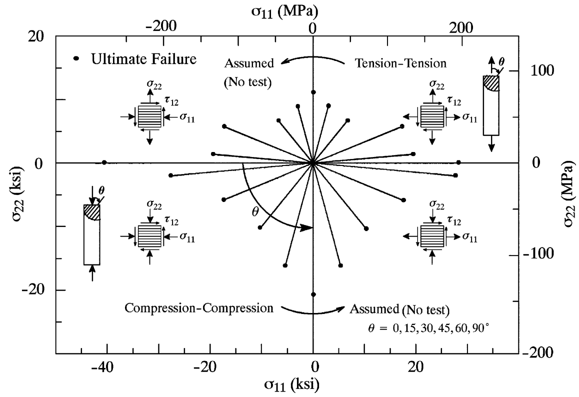

Fig. 4. Axial and transverse stress paths (σ_{11} and σ_{22}).

aligned with the loading axis. The stress and strain test data is usually in the global coupon system. The test data is transformed in the local material system using two separate in-plane stress and strain transformations. The local transformed stress and strain vectors are coupled for different loading levels to train the proposed ANN models that can generate and span all continuous paths of multi-axial stress–strain behaviors. The inelastic strain data needed for two of the ANNs was generated by subtracting the linear strain parts calculated using the orthotropic compliances of the material from the measured total strains. The compliance properties used in the later calculations for the inelastic strains were reported by El-Hajjar and Haj-Ali (2004) and shown in Table 1. The overall FVF averaged for the entire section is 0.34.

Fig. 4 shows the plane formed by the axial and transverse stress paths. The lines drawn in the different quadrants illustrate the experimental applied stress path and at what point it reached its ultimate failure filled circle. These stresses are all in the local material direction. The off-axis tension and compression tests produced tension–tension and compression–compression uni-axial and transverse stresses in addition to shear. The modified Arcan test, El-Hajjar and Haj-Ali (2004), was used in this study for pure shear mode. No tests were performed for the mixed tension–compression and compression–tension cases. These cannot be generated from the off-axis tests and may need a special multi-axial testing apparatus, which was not available. This testing limitation is overcome by using the fact that the axial stiffness is larger than the transverse. We can assume that the total strain in the tension–tension case is the same in magnitude to that of the compression tension case. This assumption can also be applied by equating the strain magnitudes of the compression–compression cases to the tension–compression strains as illustrated in Fig. 4. Figs. 5 and 6 show the axial and transverse stress paths combined with the pure shear case obtained from the modified Arcan test.

The size and type of the ANNs generated in this study are summarized in Table 2. The initial number of neurons in the two hidden layers was ten. As the training progressed and the error has not decreased, the training algorithm allows for adding more

Table 1
Material properties of FRP composites

Unit	E11	E22	G12	V12
MPa (ksi)	18,810 (2730)	11,300 (1640)	3583 (520)	0.285

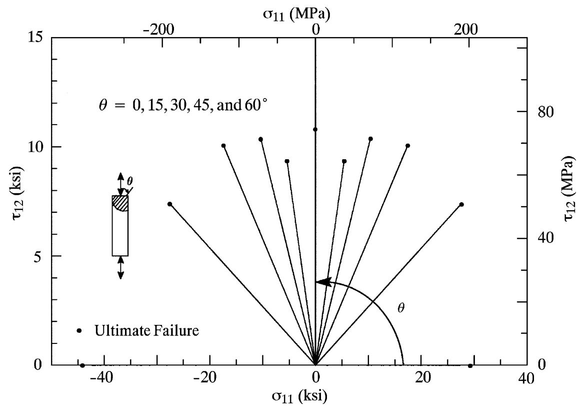

Fig. 5. Axial and shear stress paths (σ_{11} and τ_{12}) from off-axis tests. The pure shear case was obtained from the modified Arcan test.

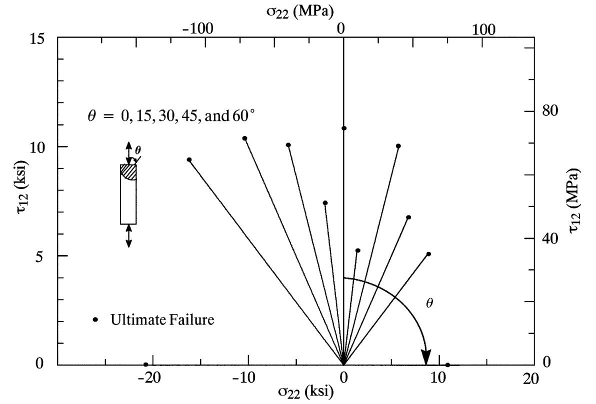

Fig. 6. Transverse and shear stress paths (σ_{22} and τ_{12}) from off-axis tests. The pure shear case was obtained from the modified Arcan test.

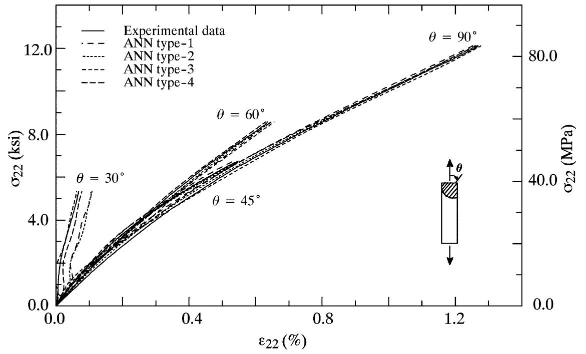

Fig. 7. ANN representation of transverse stress–strain in the local material direction after training.

Table 2
Training cases for different ANN structures

	ANN output	Hidden layer number	Initial neuron number	Final neuron number
Type 1	vector/ total	2	10	29
Type 2	vector/ inelastic	2	10	25
Type 3	scalar/ total	2	(10, 10, 10)	(26, 25, 27)
Type 4	scalar/ inelastic	2	(10, 10, 10)	(24, 23, 25)

Note that there are three ANNs for the single-variable output in cases 3 and 4.

neurons to allow for further reduction in error (or minimization of error). The range of final ANN is roughly around 26 neurons for the data set used (20,000 vectors) and for a similar specified global training error criterion (0.5%).

Next, the trained ANN models are used to generate multi-axial stress–strain behaviors and are compared with available experimental results. Fig. 7 shows the transverse tension stress–strain behaviors obtained from the trained ANNs and test results from the different off-axis coupons (i.e. $\theta = 30°$, 45°, 60° and 90°) in their local material direction. The overall responses represented by the trained ANNs are very close to the experimental responses for most of the cases except for an off-axis angle of 30°, where the error is the largest (see more discussion at the end of this section). In this case, the ANN response is directly compared against the same data that was used for training and we are examining the ability of the trained ANN to extrapolate the training data. The term "ANN prediction" is used in this paper to describe the ANN response against the effective global material behavior or as a complete multi-axial nonlinear constitute model able to predict all in-plane strain components when given the in-plane stress components. The 30° case, in Fig. 7, has the largest error compared to all other trained stress–strain paths that are not shown. The transverse stress–strain responses generated from the single-variable output ANNs (cases 3 and 4) are very close to the experimental results as that of the vector output structural ANNs (i.e. cases 1 and 2). However, it is not clear which ANN can produce the best response as the output of all of them agrees well with the experimental observations. Using the transformation of the local material (ANN) stresses and strains, the global nominal stress–strain responses are generated for each off-axis case. Fig. 8 shows the nominal off-axis tension responses expanded with the experimental results. The positive horizontal axis is used to plot the global axial strain, while the negative part is used for the transverse strain. The axial stress in the global direction is plotted against both the axial and transverse (Poisson's effect) strains. The overall transformed ANN responses are much closer to the experimental results once they are described in the global material direction. It is interesting that the proposed ANNs can generate much closer response to the experiment for the axial strain (ε_{11}) case than for the transverse strain (ε_{22}) case. In addition, the worst prediction case (i.e. local stress–strain behaviors at an off-axis angle of 30°) also produces a good agreement with experiments once they are converted into the global responses. Fig. 9 also shows the off-axis compression responses predicted by the trained ANNs and experiments. Similar to the tension responses, the global axial stress is plotted against both axial and transverse strain. In the

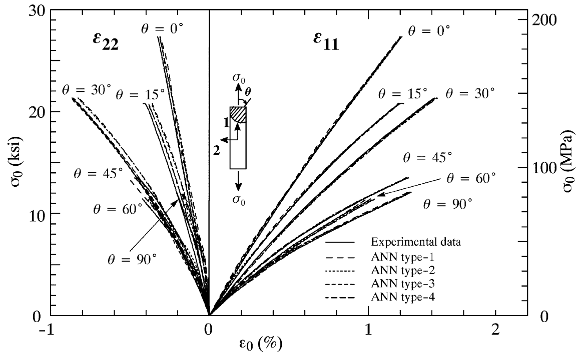

Fig. 8. Global tension stress versus direct and poisson's strains calculated from the local response of the trained ANNs.

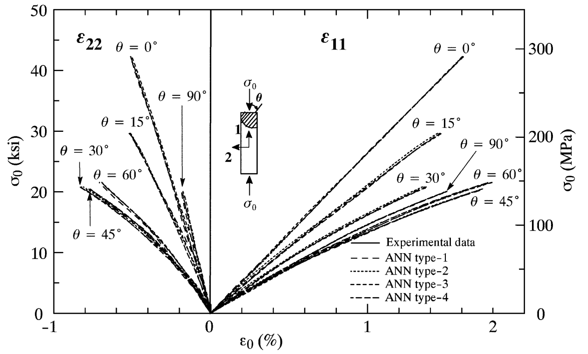

Fig. 9. Global compression stress versus direct and poisson's strains calculated from the local response of the trained ANNs.

compression case, the proposed ANNs can generate much closer responses to experiments than the tension case. As a result, it is demonstrated that the trained ANNs can generate good overall responses for all experimental cases. Good agreement with experimental data has been observed relatively more in the global responses. This can be explained by the reduced sensitivity when the stresses and strains are transferred to the global coupon-level stress–strain states. For example, this may explain why while the error is relatively larger for the local 30-degree case (Fig. 7), the combined off-axis response for this case shows good comparison in the global direction. That is, the transverse stress component (normal to the fiber) in this direction is smaller compared to the axial stress component and as such, its error is less pronounced.

4. Nonlinear finite element analysis coupled with ANN constitutive models

The trained nonlinear ANN constitutive models should be suitable for integration as nonlinear models at the Gaussian material points in a general purpose FE code. Otherwise, the generated ANN models are not complete as they provide for a limited and partial approximation in parts of the response spectrum. The trained ANNs are implemented as user-defined nonlinear material models within the FEA software. The ABAQUS general purpose FEA code is coupled with the trained ANN material module for FRP composites. The nonlinear constitutive environment in displacement-based FE typically requires the user material to determine the current stress state given the strain increment and the previous history including strain, stress, time, and other state variables. Classical inelastic mechanics models are formulated using stress variables. This was the approach taken in this study with the input to the ANN. However, since the FE environment directly supplies the displacement gradients (strains), it makes sense to generate ANNs that have strain as an input (with/without history) and stresses as output. This type of ANN has the potential of dramatically reducing the computational (iterative) effort that is required at the material level and dramatically increasing the computational efficiency. However, it should be mentioned that the uniqueness of the computed stress state must be accounted for computationally or by the structure of the ANN itself. The strain and stress-based input ANNs were coupled with the FE code, and verification of their performance was needed. Towards that goal, a notched composite plate with an open hole was tested in order to examine the simulation results from the coupled FE with ANN material models. Fig. 10 shows an FE model used in this simulation. A quarter shape of a rectangular coupon is modeled. The width and height are 0.875 in. (2.22 cm) and 6 in. (15.24 cm), respectively. The thickness is 0.5 in (1.27 cm) and the hole radius is 0.25 in. (0.635 cm). The finite element model is composed of approximately 600 nodes and 550 plane-stress type elements (CPS4R). This FE model is implemented with the ANN user material modules and the new coupled ANN–FE code was verified for simple homogeneous cases. The choice of structural modeling for a transversely oriented thick-section and notched composite plate was made because of the relatively large nonlinear responses expected. Remote uniform displacement is applied.

Fig. 11 shows the experimental set-up for the notched plate and schematic drawings of the test and FE mesh. The upper and lower parts of the cou-

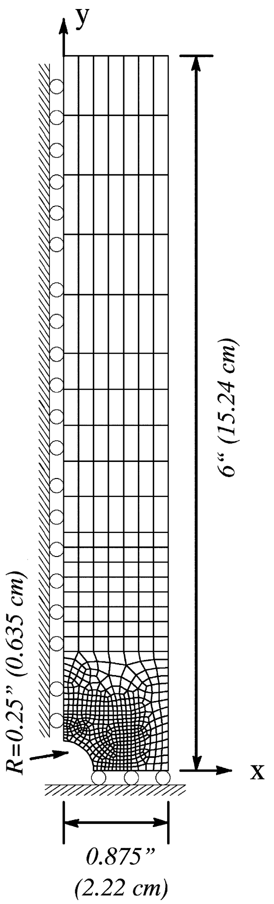

Fig. 10. Geometry of the quarter FE model with hole.

pon were gripped by the jaws of MTS-810. Monotonic tension was applied as a uniform end displacement along with relative displacement that was acquired from a 2″ extensometer attached on the specimen, as shown in Fig. 12b, and located symmetrically about the mid-plane. Fig. 12 shows the remote nominal stress versus the normalized extensometer displacement (strain) for both the FE–ANN simulation and the test. The FE–ANN coupled model is capable of predicting the overall behavior of the composite plate, while the experimental and FE–ANN extensometer response is linear. The local response around the open hole is nonlinear as shown in Fig. 13.

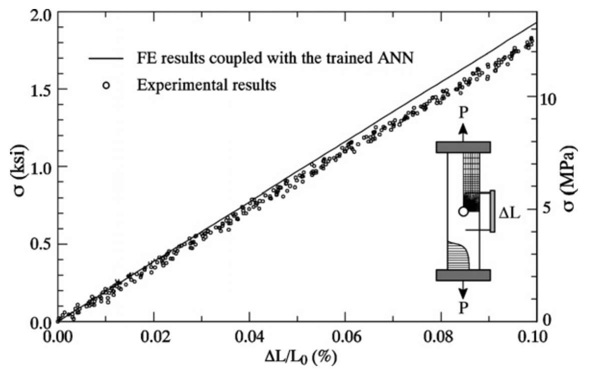

Fig. 12. Prediction for the remote stress–strain curves obtained from experiments and FE simulations implemented with the proposed ANN material model.

The FE-ANN simulations are limited to the range of the trained ANN material models that are about 2% strain in axial and transverse directions. This explains why the global response is linear, and the local response can be nonlinear. Fig. 14 schematically illustrates the unstable fluctuating ANN response beyond the training limit points. This leads to unsmooth FE convergence and ultimately divergence. The fact that the local stress has exceeded the training ANN level indicates the presence of

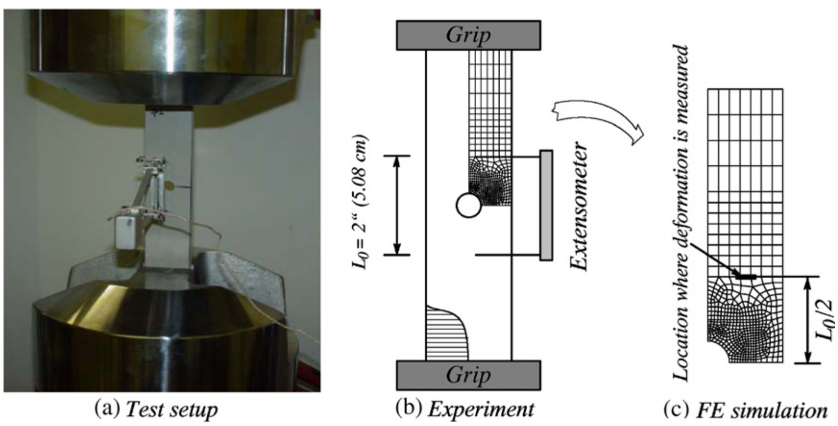

Fig. 11. Schematic drawing of the experiment and FE simulation used in the verification.

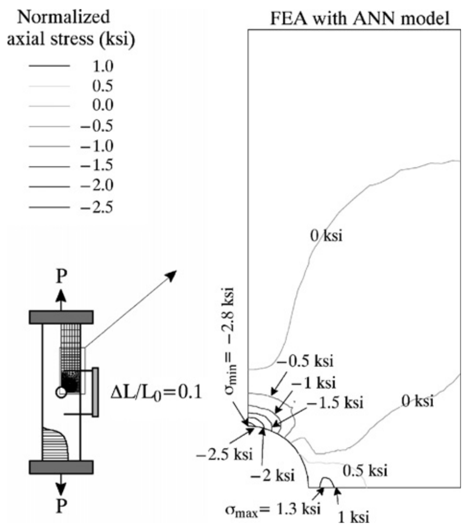

Fig. 13. Normalized axial stress contours generated from the proposed ANN material model.

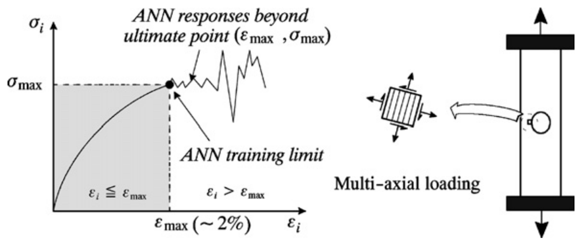

Fig. 14. Schematic drawing of the ANN response out of range of the training data.

local damage. Artificial neural network with damage capability is beyond the scope of this paper.

5. Conclusions

A new approach to generate nonlinear multi-axial ANN constitutive models for FRP composites has been demonstrated. The proposed ANN material models were trained from select experimental tests and including off-axis tension and compression along with a modified Arcan test for shear stress–strain response. The ANN material models are effective and can be constructed using the total strain or the inelastic strain parts. In addition, vector-based and single-variable output type ANNs can be used. The new constitutive models are limited within the training data range. They can be effectively implemented and coupled with FE analysis to provide for general nonlinear material response at the Gaussian integration points. The proposed FE–ANN simulation code can be used for the analysis of layered composite plate and shell structures. The fully developed nonlinear ANN constitutive models can be extended for damage and time-dependent behavior.

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