Proceedings of the 8th U.S. National Conference on Earthquake Engineering
April 18-22, 2006, San Francisco, California, USA
Paper No. 1910

EFFECT OF DESIGN VARIABLES ON DISPLACEMENT DUCTILITY OF FRP-REHABILITATED RC SQUAT COLUMNS

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ABSTRACT

The seismic performance of reinforced concrete (RC) squat columns in existing buildings or bridge piers indicates that they are vulnerable to non-ductile shear failure due to their low shear span to depth ratio. Wrapping RC columns with carbon or glass fibre-reinforced polymer (FRP) sheets proved to increase their shear capacity and consequently the flexural displacement ductility without significant increase in the column's stiffness. The displacement ductility capacity of a structural element is one of the widely used performance parameters.

A designer of a rehabilitation scheme using FRP wraps for a rectangular RC squat column should develop an understanding for the influence of different factors on the column's displacement ductility capacity. Unlike in laboratory-tested columns, designing the required number of FRP wraps to reach a targeted displacement ductility for a squat column in an existing old structure involves several uncertainties in the design variables.

A total of eleven variables that influences the design of FRP jacket are grouped into four categories, namely; material properties, geometrical properties, reinforcement content and axial load level. The study evaluates the effect of changing these variables on the displacement ductility capacity of FRP-rehabilitated RC rectangular squat columns.

Introduction

Rectangular reinforced concrete (RC) columns are widely used in bridge pier design, and they make up the majority of building columns. Columns in need of strengthening and retrofit are very common. Among those are squat columns -with low shear span to depth ratio- that are susceptible to shear failure during an extreme loading event such as a major earthquake. The traditional wisdom has been to avoid the construction of squat columns in seismically active zones. However, there exist a large number of squat columns that are at risk of brittle failure modes. These columns may have been originally designed as long columns and then partial supporting non-structural walls were later constructed therefore creating a short (squat) column. Squat columns may also have been the result of recent design following current codes.

Performance-based seismic engineering is the modern approach to earthquake resistant

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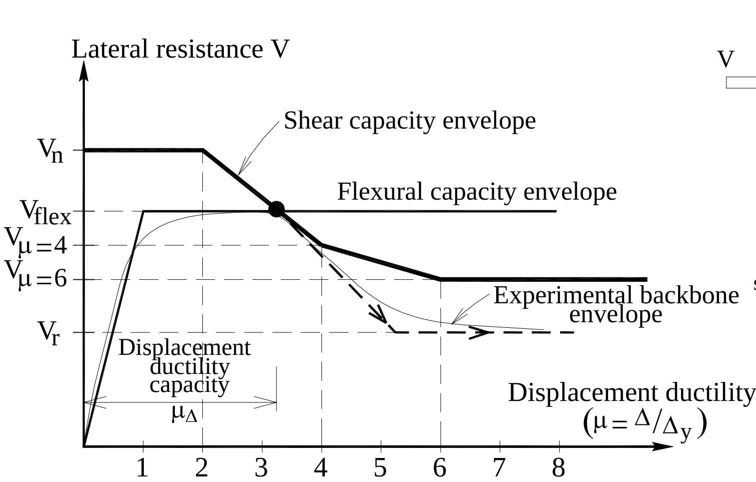
design. Seismic performance *(performance level)* is described by designating the maximum allowable damage state *(damage parameter)* for an identified seismic hazard *(hazard level)*. Performance levels describe the state of a structure after being subjected to a certain hazard level as: Fully operational, Operational, Life safe, Near collapse, or Collapse (FEMA 273/274, Vision 2000). Overall lateral deflection, ductility demand, and inter-storey drift are the most commonly used damage indices. The ductility of the column past initial steel bar yielding has become the target for good design. This approach is expected to decrease the probability of failure of the structure, and increase its energy dissipating capacity, when subjected to the design ground motions.

Fibre composites are used to increase the shear strength of existing concrete beams and columns by wrapping or partially wrapping the members. Additional shear strength contribution is introduced by orienting the fibres normal to the axis of the member or to cross potential shear cracks. Increasing the shear strength can alter the failure mode to be relatively more ductile compared to shear failure. Shear strengthening using external fibre reinforced polymer (FRP) may be provided at locations of expected plastic hinges or stress reversal for enhancing post-yield behaviour of columns subjected to seismic loads by completely wrapping the section.

A designer of a rehabilitation scheme using FRP wraps for a rectangular RC squat column should develop an understanding for the influence of different factors on the column's displacement ductility capacity. Unlike in laboratory-tested columns, designing the required number of FRP wraps to reach a targeted displacement ductility for a squat column in an existing old structure might involve several uncertainties in the design variables. The variables are grouped into four categories, namely; material properties, geometrical properties, reinforcement content and axial load level. The material properties include the compressive strength of concrete, yield stress of longitudinal and transverse rebars. The geometric properties include the ratio of the distance between longitudinal reinforcement-to-the overall depth, and the shear span-to-depth ratio of the column. The reinforcements' contents include those of the transverse reinforcement, longitudinal reinforcement and its arrangement, the FRP wrap content, and the confinement effectiveness coefficient. The level of axial load acting on the column arising from the possible variations in vertical load during a seismic event is also studied. The objective of this study is to evaluate the effect of changing these variables on the displacement ductility capacity of FRP-rehabilitated RC rectangular squat columns.

Displacement ductility capacity, μ_Δ, of FRP-rehabilitated columns

Figure 1 shows typical idealized flexural and shear lateral force-drift capacities of a reinforced concrete column. The column's response will follow the flexure envelope until it reaches the shear capacity envelope. Subsequently, degradation in the shear strength will occur and the column loses its lateral force capacity following a negative stiffness, until the residual shear capacity is reached. Several models have been developed to represent the degradation of shear strength of reinforced concrete columns with increasing deformations (Watanabe and Ichinose 1992, Ascheim and Moehle 1992, Priestly et al. 1994, Kowalsky et al. 1999, Sezen 2002, Elwood and Moehle 2005). The models do not consider columns rehabilitated with FRP wraps. Galal et al. (2005) verified a model for the shear capacity-displacement ductility envelope for axially and laterally loaded FRP-rehabilitated RC rectangular squat columns. Figure 1 shows the combined flexural and shear response of the model. The analytical predictions of the model showed reasonable correlation with the experimental results available in the literature.



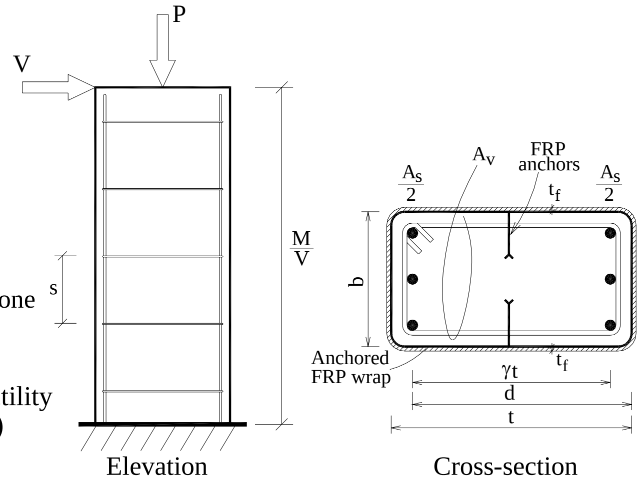


Figure 1. Shear capacity-displacement ductility model for FRP-rehabilitated columns (Galal et al. 2005).

Figure 2. FRP-rehabilitated rectangular RC subjected to lateral displacement.

In this study, the later model is used to develop the lateral resistance (V) – displacement ductility (μ) points that define the shear capacity and flexure capacity of FRP-rehabilitated RC rectangular squat columns. Given this, the displacement ductility capacity, μ_Δ, is defined as the ductility when the flexural capacity envelope intersects the shear capacity envelope. This represents the point of formation of local mechanism, which is followed by degradation in the lateral resistance of the column. The equations that were used to calculate the points identifying the shear capacity and the flexural capacity envelopes are given in the following two subsections, respectively. The equations are simplified to be expressed in the least possible number of variables. Moreover, it was found that the lateral shear resistance for both shear and flexural capacities can be expressed in terms of the cross-sectional dimensions (b.t) of the column, thus eliminating their (i.e. b.t) effect on the displacement ductility capacity of FRP-rehabilitated RC rectangular squat columns. This implies that the size of the column is assumed to have no effect on its displacement ductility capacity.

Shear capacity envelope:

The nominal shear capacity, V_n, of a RC column retrofitted with FRP composites is equal to the sum of the contributions of four mechanisms, namely; concrete V_c, axial load V_p, transverse steel V_s, and FRP V_f.

i.e. $$V_n = V_c + V_p + \min\,[\,(V_s+V_f) \text{ and } 0.66\sqrt{\beta f_c'}bd\,] \qquad (1)$$

The model assumes that the column shear capacity decreases bi-linearly with the increase of the lateral displacement ductility, μ, after reaching μ=2 such that:

$$V_{\mu=4} = \tfrac{1}{3}(V_c+V_p) + \min\,[\,(V_s+V_f) \textit{ and } 0.66\sqrt{\beta f_c'}bd\,] \qquad \text{at } \mu = 4 \qquad (2a)$$

$$V_{\mu=6} = \min\,[\,(V_s+V_f) \textit{ and } 0.66\sqrt{\beta f_c'}bd\,] \qquad \text{at } \mu = 6 \qquad (2b)$$

where f_c' is the unconfined concrete compressive strength; b is the width of the column; d is the column section depth to the tensile steel (as shown in Figure 2); β is the confined concrete compressive strength multiplier $\beta = f'_{cc}/f'_c$ and is defined later; and f'_{cc} is the confined concrete compressive strength.

The contributions of the four mechanisms reduce to:

$$V_c = 0.3\sqrt{f_{cc}'}A_e = 0.3K_e\sqrt{\beta f_c'}.bt \qquad (3)$$

$$V_p = k_p \frac{Pt/2}{H} = \frac{(\zeta f_c' bt)\cdot t/2}{2\cdot(M/Vt)\cdot t} = \frac{0.25\ \zeta\ f_c'}{M/Vt}\cdot bt \qquad (4)$$

$$V_s = \frac{A_v f_{yv} d}{s} = \rho_v f_{yv} (\frac{1+\gamma}{2})\cdot bt \qquad (5)$$

$$V_f = 0.95(2t_f)(\varepsilon_{fe} E_f)d = 1.9\rho_F \varepsilon_{fe} \frac{(1+\gamma)}{2} f_{yv}\cdot bt \qquad (6)$$

Such that $\beta = \frac{f_{cc}'}{f_c'} = 2.254\sqrt{1+7.94\frac{f_l'}{f'_c}} - 2\frac{f_l'}{f_c'} - 1.254 \geq 1.0$ (Mander et. al 1988);

$f_l' = K_e\left(\rho_v + 2\varepsilon_{fe}\rho_F\right)f_{yv}$ is the effective lateral confining pressure;

$K_e = A_e / b\ t$ is the confinement effectiveness coefficient;

$\zeta = P / (f_c'\ b\ t)$ is the axial force level;

M/Vt is the shear span-to-depth ratio of the column;

$\rho_v = A_v / bs$ is the shear stirrups content;

$\rho_F = \frac{t_f}{b}\cdot\frac{E_f}{f_{yv}}$ is the FRP wrap content; and

$\gamma = (d-d') / t$ is the ratio between longitudinal steel to the overall depth of the column t.

Where A_e is the area of effectively confined concrete core; k_p =1 for columns in double curvature and 0.5 for columns in single curvature; A_v, f_{yv}, s are the total cross sectional area, yielding strength and spacing of transverse reinforcement; $2t_f$ is the total transverse design thickness of FRP sheets (i.e. for two opposite sides); ε_{fe} is the design strain for FRP: ε_{fe}=0.004 for unanchored FRP sheets and ε_{fe}=0.006 for anchored FRP sheets; and E_f is the Young's modulus of the FRP composite material.

The above equations show that the shear capacity-to-cross sectional area, V/bt, of an FRP-rehabilitated RC squat column can be expressed as a function of eight variables, namely; f_c', K_e, ζ, f_{yv}, M/Vt, ρ_v, ρ_F, γ.

Flexural capacity envelope

In the current analysis, a bilinear flexural capacity envelope of the column is assumed. The idealized flexural envelope is defined to envelope the lateral force-displacement ductility response of the first loading cycles, in case of cyclically loaded columns. Similar bilinear idealization for un-rehabilitated RC columns has been used by several researchers (e.g. Elwood and Moehle).

Now, defining the flexural capacity envelope is reduced to determining the flexure ultimate capacity of the column, M_u, which can be easily formulated. Several researchers studied the deformational behaviour and capacity of RC columns in flexure (Mirza 1990, Panagiotakos and Fardis 2001, Biskinis et al. 2004). In this analysis, M_u is calculated by conducting section analysis considering force equilibrium and strain compatibility at failure. Equivalent rectangular stress block is assumed when considering the force equilibrium. The analysis accounts for the axial load on the column.

Therefore, the flexural capacity, M_u, of a RC column with longitudinal reinforcement content $\rho_t = A_s/bt$, where A_s is the total area of longitudinal reinforcement, can be expressed as a function of bt^2. Consequently, the shear force V_{flex} that corresponds to the column's flexural capacity can be expressed as:

$$\frac{V_{flex.}}{b \cdot t} = \frac{M_u / (M/V)}{b \cdot t} \times \frac{t}{t} = \frac{M_u}{(M/Vt) \cdot b \cdot t^2} \tag{7}$$

The above equation shows that the column's shear capacity-to-cross sectional area that corresponds to its flexural capacity, $V_{flex.}/bt$, is function of the variables that define M_u/bt^2, and these are: f_c', ρ_t, f_y, ζ, γ, in addition to the arrangement of longitudinal reinforcement in the cross-section and M/Vt.

From the formulation of the shear and flexure capacities' envelopes, it is shown that the displacement ductility capacity of FRP-rehabilitated rectangular RC squat columns depend on eleven variables that control the flexure and shear envelopes' capacities. These variables are grouped into four categories as shown in Table 1.

Table 1. Studied variables that affect the displacement ductility capacity of FRP-rehabilitated rectangular RC squat columns.

Category	Variable	Definition
Material properties	f_c'	unconfined concrete compressive strength
	f_{yv}	yielding strength of transverse reinforcement
	f_y	yielding strength of longitudinal reinforcement
Geometrical properties	γ	Ratio of the distance between longitudinal reinforcement-to-the overall depth
	M/Vt	Shear span-to-depth ratio
Reinforcements' contents	ρ_v	Shear stirrups content [$\rho_v = A_v / (b.s)$]
	ρ_F	FRP wrap content [$\rho_F = t_f E_f / (b.f_{yv})$]
	ρ_t	Longitudinal reinforcement content [$\rho_t = A_s / (b.t)$]
	K_e	Confinement effectiveness coefficient [$K_e = A_e /(b.t)$]
	a_1	percentage of the intermediate reinforcement-to-the total longitudinal reinforcement
Axial load level	ζ	Axial force level [$\zeta = P / (f_c'.b.t)$]

Maximum inelastic drift capacity Δ_m

As shown by Paulay (2001), for different reinforced concrete elements the yield displacement, Δ_y, is more or less constant, such that an increase in strength automatically results in a proportionate increase in stiffness. This implies that the maximum inelastic drift capacity, $\Delta_m = \mu_\Delta \ \Delta_y$, can be estimated if the displacement ductility capacity μ_Δ is known.

Effect of design variables on the displacement ductility–FRP content relationship

In order to study the effect of design variables on the displacement ductility of FRP-rehabilitated RC rectangular squat columns, the displacement ductility capacity (μ_Δ) – FRP content (ρ_F) relationship is considered. A typical RC rectangular squat column with specific properties as shown in Table 2 is considered; hence the effect of changing these properties on μ_Δ-ρ_F relationship is studied. The properties were chosen to represent an existing RC rectangular squat column that is designed according to pre-1970 codes (ACI 1968).

Table 2. Properties of the studied rectangular RC squat column.

Variable	f_c' MPa	f_{yv} MPa	f_y MPa	γ d/t	M/Vt	ρ_v A_v/bs	ρ_t A_s/bt	K_e	ζ $P/f_c'bt$
Value	40	400	400	0.7	1.5	0.4%	1%	0.6	0.2

The covered range of FRP content, intended to be used in the column rehabilitation, is up to ρ_F = 2%, which is equivalent to 6 layers of Carbon FRP (with laminate thickness=0.165 mm, b=300mm, E_f=240 GPa, f_{yv}=400 MPa), or 14 layers of Glass FRP (with laminate thickness=0.25 mm, b=300mm, E_f=70 GPa, f_{yv}=400 MPa).

Effect of material properties

Figure 3 shows the effect of the concrete compressive strength on μ_Δ-ρ_F relationship. From the figure, it can be seen that for a target displacement capacity, e.g. μ_Δ=5, increasing the concrete strength f_c' from 20 MPa to 50 MPa increases the required FRP content ρ_F from 0.09% to 0.42% and from 0.12% to 0.65% for anchored and unanchored FRP jackets, respectively. In other words, an underestimation of the *actual* concrete compressive strength of an existing RC column will lead to an unsafe design of the required thickness of the FRP jacket used in the column's rehabilitation to meet a specified target displacement.

Figure 4 shows the effect of the yielding strength of transverse reinforcement on μ_Δ-ρ_F relationship. From the figure, it can be seen that a RC column with f_{yv}=300 MPa (or f_{yv}=500 MPa) requires about 70% more (or 40% less) ρ_F compared to a column with f_{yv}=400 MPa. This is attributed to the fact that the reduction (or increase) in the contribution of steel transverse mechanism to the shear strength should be compensated by the contribution from the FRP mechanism in order to achieve the same target displacement ductility. It should be noted that according to the formulation of the contribution of the transverse steel and FRP mechanisms given by equations 5 and 6, respectively, the linear variation of f_{yv} for a given transverse reinforcement ρ_v does not yield a similar linear variation in the required FRP content ρ_F. From the figure, it can be seen that anchoring the FRP jacket to the column reduces the required content of FRP for a target displacement ductility, while the range of the relative increase (or decrease) in FRP content ρ_F with respect to the decrease (or increase) in f_{yv}, remains the same for anchored and unanchored FRP jackets.

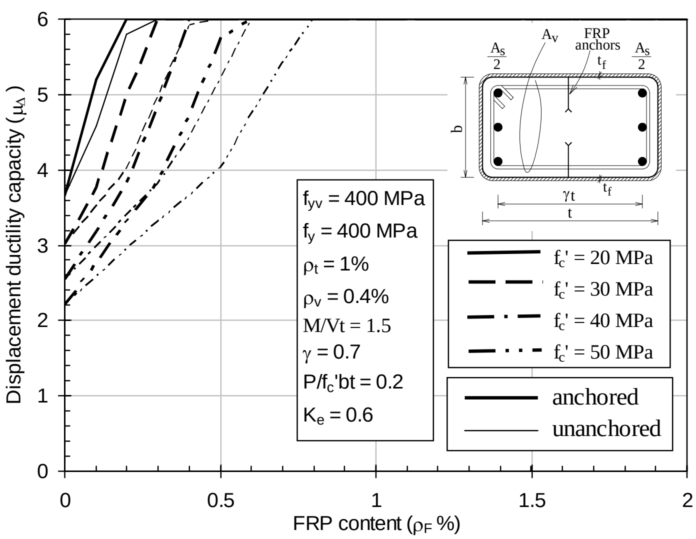


Figure 3. Effect of the concrete compressive strength on the μ_Δ-ρ_F relationship.

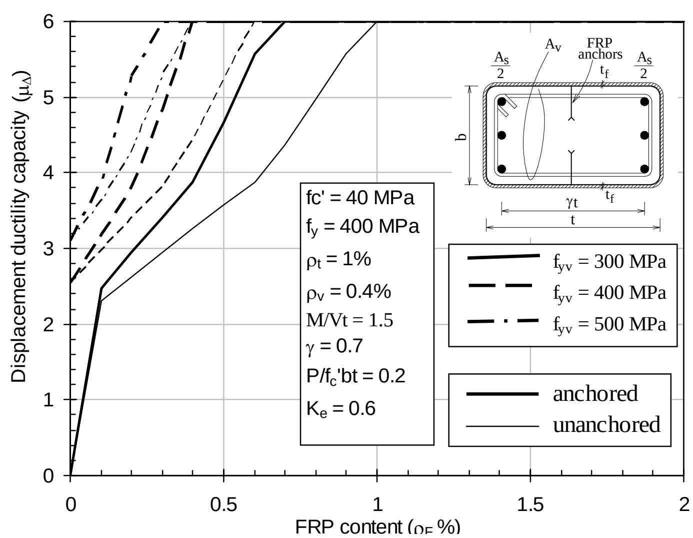


Figure 4. Effect of the yielding strength of transverse reinforcement on the μ_Δ-ρ_F relationship.

Figure 5 shows the effect of the yielding strength of longitudinal reinforcement on μ_Δ-ρ_F relationship. From the figure, it can be seen that changing the yielding strength of longitudinal reinforcement has an inverse effect on the required FRP content ρ_F at a target displacement ductility μ_Δ, compared to that of changing the yielding strength of transverse reinforcement. This is attributed to the fact that increasing the yielding strength of the longitudinal reinforcement increases its flexural capacity without affecting its shear strength capacity, which results in a reduced displacement ductility and higher required FRP content. A RC column with f_y=300 MPa (or f_y=500 MPa) requires about 35% less (or 45% more) ρ_F compared to a column with f_y=400 MPa.

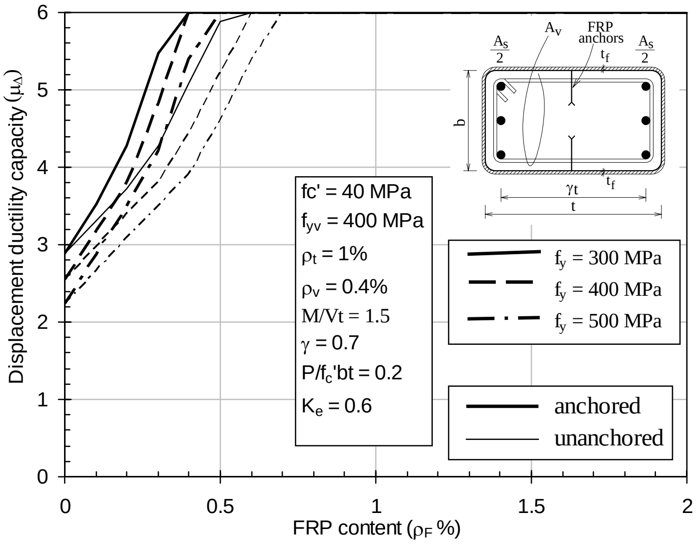


Figure 5. Effect of the yielding strength of longitudinal reinforcement on the μ_Δ-ρ_F relationship.

Effect of geometrical properties

Figure 6 shows the effect of the ratio of the distance between longitudinal reinforcement-to-the overall depth γ on μ_Δ-ρ_F relationship. From the figure, it can be seen that changing γ from 0.6 to 0.8 does not have a significant effect on the μ_Δ-ρ_F relationship for anchored and unanchored FRP-rehabilitated RC squat columns. It should be noted that this observation does not mean that γ has no effect on the flexural or shear capacities, but rather, this implies that the change in both flexure and shear capacities is such that the displacement ductility and the required FRP content of the rehabilitated column are almost the same.

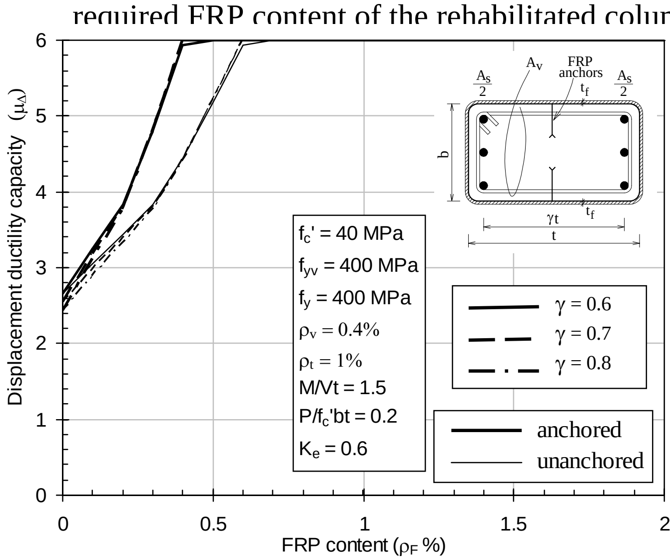


Figure 6. Effect of the ratio of the distance between longitudinal reinforcement-to-the overall depth on the μ_Δ-ρ_F relationship.

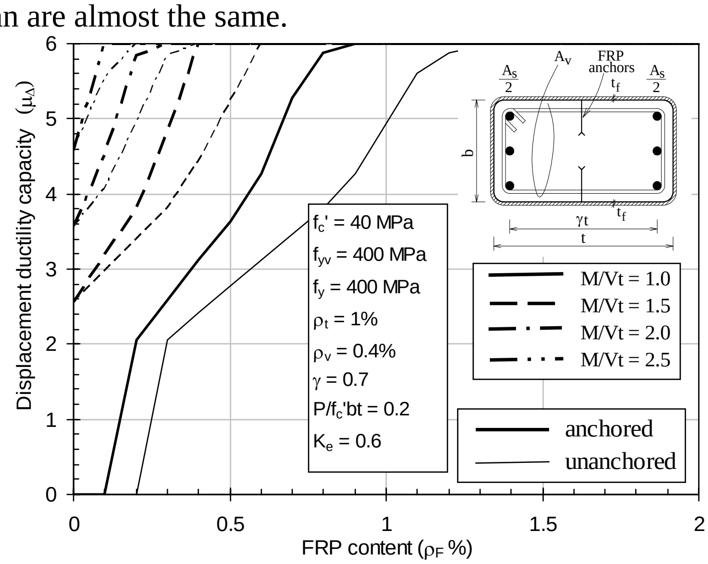


Figure 7. Effect of shear span-to-depth ratio on the μ_Δ-ρ_F relationship.

Figure 7 shows the effect of the shear span-to-depth ratio on μ_Δ-ρ_F relationship. From the figure, it can be seen that reducing the shear span-to-depth ratio of RC squat columns reduces its displacement ductility capacity and increases the required FRP content of the FRP jacket. For a targeted displacement ductility capacity μ_Δ=5 for the studied column, decreasing the shear span-to-depth ratio from 2.5 to 1.0 increases the required FRP content from $\rho_F \approx 0\%$ to $\rho_F = 0.68\%$ and 1.0% for anchored and unanchored FRP jackets, respectively. This emphasizes the importance of identifying the expected shear span of a RC column due to its impact on the required thickness of the FRP jacket, especially in the case of captive columns (for example, those created due to window openings in partially masonry-infilled frames and at the top and bottom ends of the columns in the case of masonry-infilled RC frames).

Effect of reinforcement content

Figure 8 shows the effect of transverse reinforcement content on μ_Δ-ρ_F relationship. As expected, increasing the transverse reinforcement content increases the ductility and reduces the required FRP content. This is attributed to the increase in the contribution of the transverse reinforcement mechanism and thus the shear capacity. For the studied column, anchoring the FRP jacket to the column reduces the required content of FRP by about 35% for a transverse reinforcement content range of 0.2% to 0.8%.

Figure 9 shows the effect of the longitudinal reinforcement content on μ_Δ-ρ_F relationship. Increasing the longitudinal reinforcement content decreases the displacement ductility and increases the required FRP content. This is attributed to the fact that increasing the longitudinal reinforcement content increases the flexural capacity, which in turn reduces the displacement ductility capacity. For example, for a target displacement ductility capacity μ_Δ=5 for the studied column, an underestimation of the longitudinal reinforcement content of the existing column as 0.5% rather than 1%, results in an underestimation of the required FRP content by 40%. Yet, anchoring the FRP jacket diminishes this underestimation.

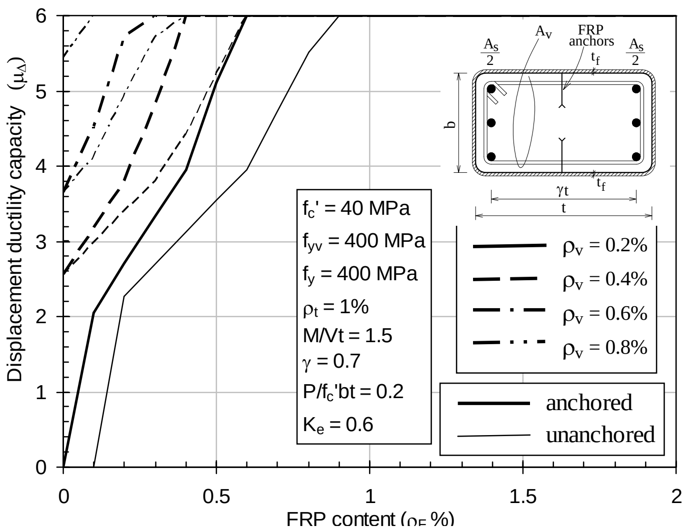


Figure 8. Effect of transverse reinforcement content on the μ_Δ-ρ_F relationship.

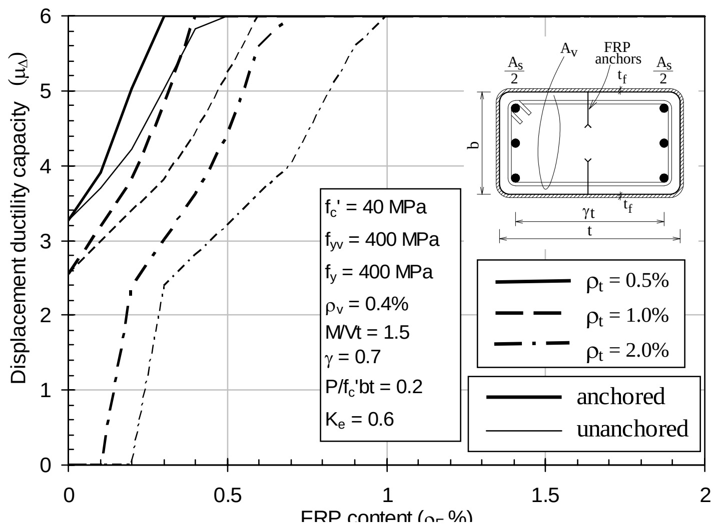


Figure 9. Effect of longitudinal reinforcement content on the μ_Δ-ρ_F relationship.

Figure 10 shows the effect of the confinement coefficient K_e on μ_Δ-ρ_F relationship. For the studied column, increasing the confinement coefficient increases the displacement ductility capacity and reduces the required FRP content. This effect diminishes as the targeted displacement ductility capacity reaches μ_Δ=6. On the other hand, a column with anchored FRP jacket will have higher displacement ductility capacity and less required FRP content compared to unanchored one. The effect of changing the transverse reinforcement content ρ_v to 0.2% and 0.4% for K_e=0.2 is shown on the same figure, where it can be seen that decreasing ρ_v decreases the displacement ductility capacity and increases the required content of FRP.

Figure 11 shows the effect of the arrangement of the longitudinal reinforcement on μ_Δ-ρ_F relationship. From the figure, it can be seen that increasing the percentage of the intermediate reinforcement-to-the total longitudinal reinforcement in the cross section, a_1, increases the displacement ductility and decreases the required FRP content. This is attributed to the fact that increasing this percentage reduces the flexure capacity of the section thus increasing the displacement ductility.

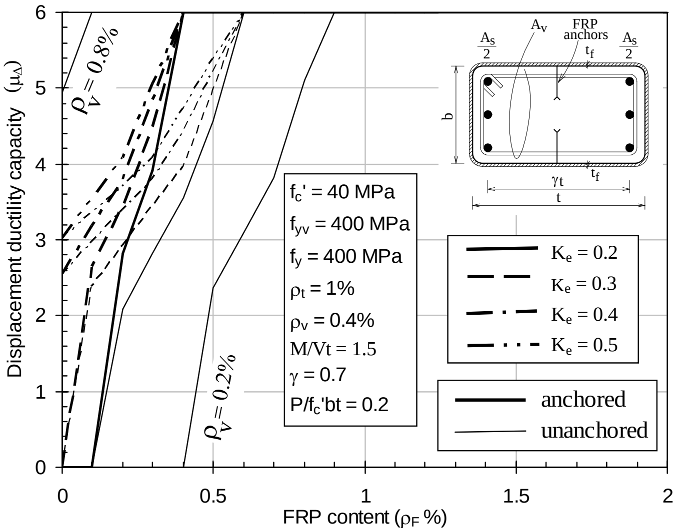


Figure 10. Effect of the confinement coefficient K_e on the μ_Δ-ρ_F relationship.

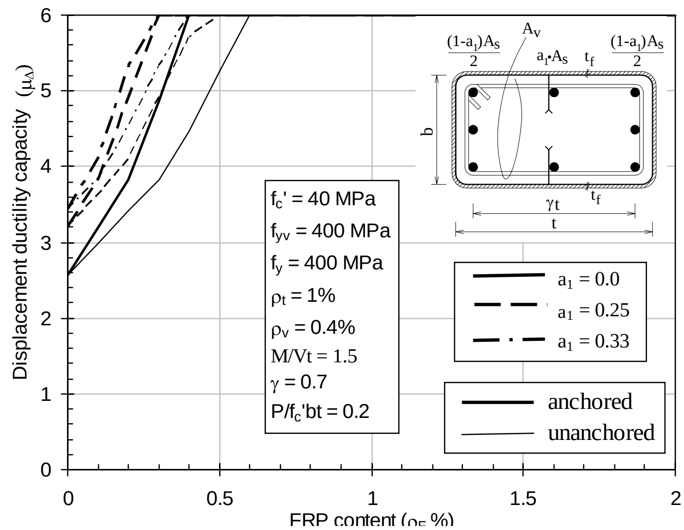


Figure 11. Effect of the longitudinal bar arrangement on the μ_Δ-ρ_F relationship.

Effect of axial load level

Figure 12 shows the effect of the axial load level on μ_Δ-ρ_F relationship. From the figure it can be seen that, for a target displacement ductility capacity μ_Δ=5 for the studied column, increasing the axial load level from $0.1P_o$ (where $P_o=f_c'bt$) to $0.3P_o$ increases the required FRP content from 0.1% to 0.65%. Further increase in the axial load from $0.3P_o$ to $0.4P_o$ does not change the required FRP content. For the studied column, anchoring the FRP jacket to the column reduces the required content of FRP ρ_F by about 40%.

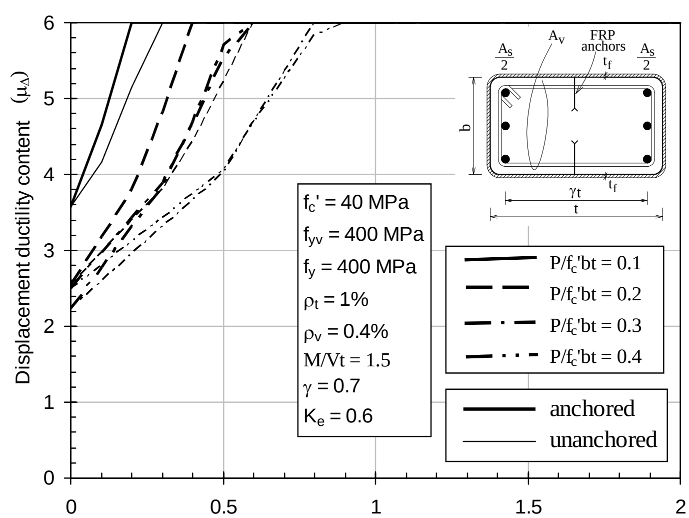


Figure 12. Effect of axial load on the μ_Δ-ρ_F relationship.

Conclusions

The effect of eleven variables that control the flexure and shear capacities and thus the displacement ductility and drift capacities of FRP-rehabilitated rectangular RC squat columns, is evaluated. The formulation of the flexure and shear capacities is based on an existing model. The variables are grouped into four categories, namely; material properties, geometrical properties, reinforcement content and axial load level. In order to study the effect of these design variables on the displacement ductility of FRP-rehabilitated RC rectangular squat columns, the displacement ductility capacity (μ_{Δ}) – FRP content (ρ_F) relationship is considered. Based on the analytical study, the following conclusions regarding the rehabilitation of existing RC rectangular columns using FRP wraps are reached:

1- For a target displacement ductility capacity, the required FRP content of the FRP jacket increases with the increase of concrete compressive strength, yielding strength of longitudinal reinforcement, longitudinal reinforcement content, and axial load level.
2- For a target displacement ductility capacity, the required FRP content of the FRP jacket increases with the decrease of yielding strength of transverse reinforcement, shear span-to-depth ratio, transverse reinforcement content, confinement effectiveness coefficient, and percentage of intermediate longitudinal reinforcement.

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