Delamination of laminated fiber reinforced plastic composites under multiple cylindrical impact

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Received 7 June 2005; accepted 24 January 2006
Available online 27 March 2006

Abstract

In the present paper a 3D finite element analysis has been performed for assessing delamination at the interfaces of graphite/epoxy laminated fiber reinforced plastic composites subjected to low velocity impact of multiple cylindrical impactors. Eight nodded layered solid elements have been used for the finite element analysis of fiber reinforced plastic laminates. Newmark-β method along with Hertzian contact law has been used for transient dynamic finite element analysis and an algorithm has been developed for determining the response of the laminated plate under the multiple impacts at different time. Appropriate delamination criterion has been used to assess the location and extent of delamination due to multiple impacts. A study has been carried out to observe the effects of important parameters on the impact response of the laminate and the delamination induced at the interfaces. It has been observed that the contact force magnitude as well as delamination at the interface are greatly influenced by the time interval between successive multiple impacts.

Keywords: Delamination; Graphite/epoxy; Impact; Multiple impacts; Finite element method

1. Introduction

Fiber reinforced plastic (FRP) laminated composites has been extensively used in aerospace and allied industries due to their inherent advantages like high strength to stiffness ratio. In spite of having these advantages, these materials are also susceptible to damages under transverse impacts and the nature of damage induced due to such impacts are entirely different than that in case of conventional metallic materials. In laminated FRP components, the damage mode usually consists of local permanent deformations, fiber breakage, delamination, matrix cracking, etc. Especially, in the case of impact low velocity impact, the resulting damages like, delamination occurs at specific interfaces of the laminate. These defects are sub surface in nature and cause considerable reduction in structural stiffness leading to growth of the damage and final fracture.

Therefore, the impact response of laminated FRP composites has been an important area of research for long time. A large number of analytical as well as experimental works have already been reported in literature in this direction and some of the important works are discussed here.

Sun and Chattopadhyay [1] studied the contact force history of a simply supported laminate with initial stress subjected to impact loading by solving a non-linear integral equation. Sun and Chen [2] studied the impact response of initially stressed laminate using a 2D finite element analysis and reported the effects of impactor velocity, impactor mass and the initial stress on the impact response of the laminate. Wu and Chang [3] performed transient dynamic finite element analysis of laminated FRP plate subjected to impact of foreign objects and presented the stress and strain distribution through the laminate thickness during the impact. Choi and Chang [4,5] developed a model for damage initiation and the extent of damage as a function of material property, laminate configuration and impactor mass. Lee et al. [6] studied the impact response of hybrid-laminated composites under low velocity impact. Choi
and Hong [7] studied the frequency response of impact force history from modal analysis and compared the same with the natural frequency of the system where the mass of the impactor was lumped to the plate. Goo and Kim [8] studied the impact behavior of curved composite plates using penalty finite element method. Johnson et al. [9] developed a continuum damage mechanics model for studying the impact response and the delamination due to impact of a steel ball on a carbon/epoxy laminate. Guinard et al. [10] studied the localized damage due to transverse impact using a damage meso model for low velocity impact of laminated plates. Sung et al. [11] used acoustic emission along with wavelet transformation to detect matrix cracks and free edge delamination in graphite/epoxy laminates. Luo et al. [12] presented an approach for evaluation of impact damage initiation and propagation in composite plates using stress based delamination criterion.

McLaughlin and Santhanam [13] developed a 2D finite element model for simulating damage growth in cross ply symmetric laminates. Li et al. [14] developed a finite element based model for simulating low velocity impact induced damage in laminated composites and also used adaptive finite element analysis for increasing computational efficiency of the model. Duan and Ye [15] developed a 3D finite element model incorporating frictional contact for studying the delamination at the interfaces due to low velocity impact and showed excellent agreement with experimental results. Moura and Marques [16] performed numerical analysis and also conducted experiments to predict damages in carbon epoxy laminates subjected to low velocity impact. Zou et al. [17] developed a continuum damage model to study the delamination at the interfaces between constituent layers due to multiple impacts have been studied. Delaminations at interfaces between the constituent layers due to multiple impacts have been studied using the present code.

3. Finite element formulation

3.1. 8-Noded layered solid element

Three dimensional 8-noded isoparametric layered solid element was used for full 3D modeling of the laminated plate (Fig. 2). The shape function defining the geometry and displacement are
\[ N_i = \frac{1}{2}(1 + rr_i)(1 + ss_i)(1 + tt_i) \quad i = 1, 2, 3, \ldots, 8 \quad (1) \]

where \( r, s, t \) are natural coordinates and \( r_i, s_i, t_i \) are the values of natural coordinates for the \( i \)th node. In order to simulate the flexural response, extra shape functions are introduced to the general 8-noded solid element and they are

\[ P_1 = (1 - r^2) \quad P_2 = (1 - s^2) \quad P_3 = (1 - t^2) \quad (2) \]

The displacement variation within the element is given by

\[
\{d\} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^{8} N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} + \{P\} \{\Psi\} \quad (3)
\]

\[
\begin{bmatrix} P_1 & P_2 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_1 & P_2 & P_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_1 & P_2 & P_3 \end{bmatrix}
\]
and
\[ [\Psi]^T = [\Psi_1 \Psi_2 \Psi_3 \Psi_4 \Psi_5 \Psi_6 \Psi_7 \Psi_8 \Psi_9] \]
The stiffness matrix is calculated as
\[ [K] = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} [B]^T [C] [B] [J] \, dr \, ds \, dt \]
(4)

Let
\[ G(r,s,t) = [B]^T [C] [B] [J] \]
then
\[ [K] = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} G(r,s,t) \, dr \, ds \, dt \]
(6)

For an element having \( N \) layers in the thickness direction (Fig. 2),
\[ T = \sum_{k=1}^{N} t_k \]
(7)
where \( T \) is the total thickness of the element and \( t_k \) is the thickness of the \( k \)th layer of the element. Taking a parameter \( t \in [0,T] \) and changing the limits
\[ [K] = \frac{2}{T} \int_{-1}^{1} \int_{-1}^{1} \sum_{k=1}^{N} \frac{t_k - t_{k-1}}{2} \int_{-1}^{1} F(r,s,t) \, dr \, ds \, dt \]
(8)
where \( t \) is function of thickness over the layer.

2 \times 2 \times 2 Gauss quadrature scheme is applied to evaluate the above integration. The stiffness matrix thus evaluated by Eq. (8) is of size 33 \times 33 and includes coefficients pertaining to the incompatible modes. Using static condensation technique these terms are eliminated and the condensed stiffness matrix becomes of the order of 24 \times 24 pertaining to nodal degrees of freedom only. Element mass matrix is evaluated as
\[ [M^T] = \int_{\Omega} [N]^T \rho [N] \, dv \]
(9)

### 3.2. Finite element contact impact modeling

Dynamic equation governing the impact problem (neglecting damping) at time \( t + \Delta t \) is
\[ [M] \{\ddot{d}\}^{t+\Delta t} + [K] \{d\}^{t+\Delta t} = \{F\}^{t+\Delta t} \]
(10)
where \([M]\) and \([K]\) are the mass and stiffness matrices and \([F], \{d\}, \{\dot{d}\}, \{\ddot{d}\}\) are force, displacement, velocity and acceleration vectors for plate, respectively, at time \( t + \Delta t \).
Using Newmark-\( \beta \)-method, Eq. (10) can be reduced to
\[ [\tilde{K}] \{\ddot{d}\}_U^{t+\Delta t} = \{\tilde{F}\}_U^{t+\Delta t} \]
(11)
where \([\tilde{K}]\) is the effective stiffness matrix and \([\tilde{F}]\) is the effective force vector and are defined as
\[ [\tilde{K}] = \frac{1}{\phi \Delta t^2} [M] + [K] \]
(12)
\[ \{\tilde{F}\} = \{H\}' + \{F\}^{t+\Delta t} \]
(13)

\[ \{H\}' = [M] \left( \frac{1}{\phi \Delta t^2} \{d\}' + \frac{1}{\phi \Delta t} \{\dot{d}\} + \left( \frac{1}{2 \phi} - 1 \right) \{\ddot{d}\} \right) \]
(14)
where \( \phi \) and \( \delta \) are the Newmark constants.

In Eq. (11), displacement, velocity and acceleration at time \( t \) are known at each point inside the plate and the unknown quantities in this equation are vector \([\ddot{d}]^{t+\Delta t}\) and the force vector \([F]^{t+\Delta t}\). In the absence of pre-load, Eq. (11) becomes
\[ [\tilde{K}] \{d\}^{t+\Delta t} = \{H\}' + \{P\}^{t+\Delta t} \]
(15)
where \([P]\) is the contact force.

The displacement vector \([d]\) is expressed as the sum of the displacements due to the force \([H]\) and the contact force \([P]\) as
\[ \{d\}^{t+\Delta t} = \{d\}^t_{U} + \{d\}^t_{P} \]
(16)
Eqs. (15) and (16) give
\[ [\tilde{K}] \{d\}^{t+\Delta t}_{U} = \{H\}' \]
(17)
and
\[ [\tilde{K}] \{d\}^{t+\Delta t}_{P} = \{P\}^{t+\Delta t} \]
(18)
Writing \([P]\) \( f^{t+\Delta t} \{U\} \), where \( f^{t+\Delta t} \) is the magnitude of contact force at time \( t + \Delta t \). Eq. (18) yields
\[ [\tilde{K}] \{d\}^{t+\Delta t}_{U} = f^{t+\Delta t} \{U\} \]
(19)
and for a unit contact force (\( f^{t+\Delta t} = 1 \))
\[ [\tilde{K}] \{d\}^{t+\Delta t}_{U} = \{U\} \]
(20)
where \([d]^{t+\Delta t}_{U}\) is the displacement caused by the unit contact force and
\[ \{d\}^{t+\Delta t}_{P} = f^{t+\Delta t} \{d\}^{t+\Delta t}_{U} \]
(21)
Eqs. (17) and (21) give
\[ \{d\}^{t+\Delta t} = \{d\}^{t+\Delta t}_{U} + f^{t+\Delta t} \{d\}^{t+\Delta t}_{U} \]
(22)

#### 3.2.1. Contact of a cylinder with a plate

When a plate is impacted by a mass, the magnitude of contact force, which results because of the impact, is not known a priori. This contact force needs to be calculated before the plate motion is analysed. The evaluation of contact force depends on a contact law, which relates the contact force with the indentation.

A Hertzian distribution of pressure is assumed to act on plane, which is uniform along the length of barreled cylinder. In order to reduce stress concentration at ends, the axial profile of the cylinder should be slightly barreled. The contact force is given by [26]
\[ f = \frac{\pi l E' \delta}{1.886 + \ln(l/a)} \]
(23)
where \( \delta \) is displacement at the center of contact. \( 2l \) is the length of the cylinder in contact and \( a \) is given by
where, \( r \) is radius of the cylinder, \( E_i \) and \( E \) are Young’s moduli of impactor and target normal to the fiber direction in the uppermost composite layer, respectively. \( \nu_i \) and \( \nu \) are Poisson’s ratios of impactor and target, respectively. Eq. (23) is considered as the relationship between indentation force at \( t \) and \( d \). At time \( t + \Delta t \) the contact force is obtained

\[
C = \frac{4f r}{\pi E} + \frac{1}{E} - \frac{1}{E_i} + \frac{1}{E} - \frac{1}{E_i} \tag{24}
\]

At time \( t + \Delta t \) this depth is

\[
\delta_{z_{i+1}} = \delta_{z_i} + \frac{1}{E_i} + \frac{1}{E} - \frac{1}{E_i} \frac{f t}{m} \tag{25}
\]

Using Eq. (25)

\[
\delta_{C} = \frac{\delta_{z_{i+1}} - \delta_{z_i}}{\delta_{z_{i+1}} + \delta_{z_i}} \tag{26}
\]

Combining Eqs. (25)–(30) the following expressions for the contact force are obtained

\[
f_{t+\Delta t} = \frac{\pi E}{1386 + \ln(l/a)} \left( \int_0^{t+\Delta t} \frac{f t}{m} \, dt \, dr \right) - \delta_{C} = \frac{\delta_{z_{i+1}} - \delta_{z_i}}{\delta_{z_{i+1}} + \delta_{z_i}} \tag{27}
\]

Contact force at \( t + \Delta t \), i.e. \( f_{t+\Delta t} \) was calculated using Eq. (28) (during loading or during unloading) by Newton Raphson method. From the known value of contact force \( f_{t+\Delta t} \), displacement vector \( \{d\}_{t+\Delta t} \) is calculated using Eq. (22). Once the value of \( f_{t+\Delta t} \) is known, plate velocity, acceleration and then the stresses and strains at time \( t + \Delta t \) were calculated. This procedure has been repeated for each time step to get the displacement, stress and strain.

3.2.2. Stress based criterion for delamination

In order to assess delamination initiation at the interface of the laminate, the criterion proposed by Choi et al. [4,5] for impact induced delamination has been used in the present work. The criterion is

\[
D_a \left[ \left( \frac{\sigma_{n+1}}{S_{n+1}} \right)^2 + \left( \frac{\sigma_{n+1}}{S_{n+1}} \right)^2 + \left( \frac{\sigma_{n+1}}{S_{n+1}} \right)^2 \right] = e_d^2 \tag{29}
\]

where

\[
e_d \geq 1 \text{ failure} \quad e_d < 1 \text{ no failure} \tag{30}
\]

and \( D_a \) is an empirical constant determined from experiment, which has been taken as 1.8 as suggested by Choi et al. [4]. \( \sigma_{n+1}, \sigma_{n+1} \) and \( \sigma_{n+1} \) are average stresses in the interface between \( nth \) and \( n+1 \)th ply, respectively, and are expressed as

\[
\left[ \sigma_{n+1} \right] = \frac{1}{h_{n+1}} \int_{h_{n+1}}^{h_{n+1}} [\sigma_{n+1}] \, dt \tag{31}
\]

3.2.3. Multiple impact algorithm

Based on the formulation described above, analysis of multiple cylindrical impacts on the FRP plate has been implemented as follows:

Define time step \( \Delta t \)

For each impact \( i \):

- Solve for \( \{d_i\} \) from \( \{d_i\}_{t+\Delta t} = \{U_i\} \)
- \( \{d_i\} \) is displacement due to unit load at contact point in impact direction

![Fig. 3a. Contact force history for a steel ball impacting on an isotropic plate.](image-url)
For each time step:

1. Calculate \( \{H\}' = \{M\} \{a_0 + a_2\tilde{d} + a_3\tilde{d}'\} \).
2. Solve for \( d_H \) from \( \{K\} \{d\}' = \{H\}' \).
3. For each impact \( i \):
   - If (impact time < time at time step)
     Calculate contact force \( f^{i+\Delta t}_i \) using Eq. (28)
   - Else
     Contact force \( f^{i+\Delta t}_i = 0 \)
4. Calculate displacement using
   \[
   \{d\}'^{i+\Delta t} = \{d\}_{H}^{i+\Delta t} + \sum_{i=1}^{N} f^{i+\Delta t}_i \{d_p\}_i ,
   \]
   where \( N \) number of impacts
5. Calculate acceleration and velocity using
   
   \[
   \{\dot{d}\}'^{i+\Delta t} = f_0 \left( \{d\}'^{i+\Delta t} - a_2\{d\}' - a_3\tilde{d}' \right)
   \]
   \[
   \{\ddot{d}\}'^{i+\Delta t} = \{\dot{d}\}' + a_0 \{d\}' + a_3 \{\dot{d}\}'^{i+\Delta t}
   \]
6. Calculate displacement due to incompatible modes.
7. Calculate strains and stresses.
8. Apply failure criteria to assess delamination.

Go to next time step

### Table 1

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ply thickness, ( h )</td>
<td>1.5875 \times 10^{-4} m</td>
</tr>
<tr>
<td>Density, ( \rho )</td>
<td>1535.68 Kg/m³</td>
</tr>
<tr>
<td>Longitudinal Young’s modulus, ( E_{xx} )</td>
<td>1.454 \times 10^{11} N/m²</td>
</tr>
<tr>
<td>Transverse, Young’s modulus, ( E_{yy} )</td>
<td>9.99 \times 10^{7} N/m²</td>
</tr>
<tr>
<td>Shear modulus in x–y direction, ( G_{xy} )</td>
<td>5.68845 \times 10^{9} N/m²</td>
</tr>
<tr>
<td>Poisson’s ratio in x–y direction, ( \nu_{xy} )</td>
<td>0.3</td>
</tr>
<tr>
<td>Poisson’s ratio in y–z direction, ( \nu_{yz} )</td>
<td>0.3</td>
</tr>
<tr>
<td>Longitudinal tensile strength, ( Y_{LT} )</td>
<td>1.778 \times 10^{9} N/m²</td>
</tr>
<tr>
<td>Longitudinal compressive strength, ( Y_{LC} )</td>
<td>1.731 \times 10^{9} N/m²</td>
</tr>
<tr>
<td>Transverse tensile strength, ( Y_{TN} )</td>
<td>5.520 \times 10^{7} N/m²</td>
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<tr>
<td>Transverse compressive strength, ( Y_{TC} )</td>
<td>2.940 \times 10^{8} N/m²</td>
</tr>
<tr>
<td>Interlaminar shear strength, ( S_{ij} )</td>
<td>1.012 \times 10^{8} N/m²</td>
</tr>
</tbody>
</table>

### 4. Results and discussion

#### 4.1. Computer code and validation

Based on the analytical model described above, a computer code has been developed in C language for
finite element contact impact analysis of a FRP laminate. The FE code has been verified by comparing the results obtained from the present code for a steel ball impacting on a steel plate with the analytical solution of Karas [27] and excellent agreements have been obtained. Figs. 3a and 3b show the comparison of contact force history and the plate and impactor displacement history results obtained from the present code and the solution of Karas.

4.2. Two cylindrical impactors striking at different time

A laminated plate made of T300/934 graphite/epoxy with ply orientation of $[0/\ldots 45/90]_{2S}$ is clamped along all the four edges and is impacted by two cylindrical impactors which strike at equally offset locations from the ends and are symmetrically located. Table 1 shows the mechanical properties of T300/934 graphite epoxy. Plate and impactor geometry and other configurations are as follows:
Dimensions of plate $0.0762 \times 0.0762 \times 0.00254$ m
Diameter of cylindrical impactors $0.0050$ m
Length of the cylinder $0.020$ m
Two locations of contact $(0.01905, 0.0381, 0.00254)$ and $(0.57150, 0.0381, 0.00254)$
Time step $1$ µs
FEM mesh $20 \times 20 \times 2$

In this case, the two impactors strike the plate at a time interval for $\Delta t$ seconds between them i.e. the second impactor strikes the plate after a definite time after the first impact. In the present work, three cases of time interval has been considered viz. for $\Delta t = 25$, 50 and 100 µs to study the effect of time interval between successive impacts on the plate response. The velocity of the impactors has been taken as 9 m/s. Fig. 4a shows contact force history when second impactor hits the plate of 25 µs after the first impact. In this figure as well as in all the subsequent figures results are plotted up to 150 µs, even though the code has been run for 300 µs. Since in all the cases studied here, contact has been observed to completely lost after 150 µs, for clarity of figures, all the histories (force, displacement, velocity) have been plot up to 150 µs. It has been observed from the figure that the first impactor comes in contact only once and the second impactor comes in contact three times before the contact is completely lost. Figs. 4b and 4c
show the plate and impactor displacement histories and it could be observed that the plate displacement at the second contact point is more compared to that of first contact point. Figs. 4d and 4e show the velocity histories of the plate and the impactor, respectively. As expected the plate velocity is more at the second contact point compared to that at the first contact point. Also, due to reloading of the second impactor the rebounding velocity of the second impactor is more compared to that of the first impactor. Figs. 5a and 5b show the contact force histories for successive impacts for time intervals of 50 and 100 μs, respectively. Comparing the contact force histories of the three cases of increasing time intervals (Figs. 4a, 5a and 5b), it could be observed that as the time interval between two successive impacts increases, the magnitude of contact force for the second impactor changes. In the present case, the magnitude of the contact force due to the second impactor is less when Δt = 50 μs compared to that in case of Δt = 25 μs. However, the contact force magnitude is much more in the case of Δt = 100 μs. This is due to the fact that the second contact point of the plate and the impactor were moving opposite to each other at the time of contact in the case of Δt = 25 μs and Δt = 100 μs leading to higher magnitude of contact force at the impact point. However in the case of Δt = 50 μs, both the contact point and the impactor were moving in the same direction leading to comparatively lower magnitude of contact force. This could be clearly observed from Figs. 6a, 6b and 6c showing the displacement of the second impactor and the corresponding contact point of the plate. Fig. 7 shows the delaminations at the three interfaces viz. 1, 2 and 3 for two successive impacts for three cases of time interval.

Fig. 6b. Displacements of second impactor and the corresponding contact point of the [0/−45/45/90]_{2s} graphite/epoxy plate.

Fig. 6c. Displacements of second impactor and the corresponding contact point of the [0/−45/45/90]_{2s} graphite/epoxy plate.
between the successive impacts (for $\Delta t = 25 \mu s$, $\Delta t = 50 \mu s$ and $\Delta t = 100 \mu s$). It could be observed that the direction of delamination is dependent on the fiber orientation of the adjacent ply, which is an already established result [5]. This is more visible in the case of two successive impacts with $\Delta t = 100 \mu s$. In all the three cases studied here, both the impactors strike with a velocity of 9 m/s. It has been observed that the shape of the delamination is not symmetric as expected due to impacts at different time. It could also be observed from the figures that for less time interval between two successive impacts, two delaminations starting from the two impact locations coalesce into one big delamination. However, as the time interval between the successive impacts increases, the delaminations remain as two distinct delaminations and do not coalesce into one. This is due to the fact that, in the case of larger time interval between the two successive impacts, contact at the first impact site is lost before the second contact begins which is clear from the contact force histories for different $\Delta t$ values. Therefore in the case of large $\Delta t = 100 \mu s$, the chances of further delaminations near the first contact point is less. Hence the two delaminations do not coalesce into one big delamination. This, however, will also depend upon the distance between the two impact points. It has been observed that the extent of delamination is more when the interval between successive impacts is less. Fig. 8 shows how the delamination at interface 2 grows with time. Before the second impactor strikes, delamination is limited to the zone surrounding the fist impact point only. As the second impact takes place,

Fig. 7. Delamination at the interfaces 1, 2 and 3 due to two successive impacts on $[0/-45/45/90]_{2s}$ graphite/epoxy plate at different time interval between two impacts with a velocity of 9 m/s.
delamination starts growing at that location also and at the end, two delaminations coalesce to form one single delamination. However whether delamination will grow into one single delamination or remain two distinct delamination depends upon the time interval between the two impacts as well as the distance between the two impact points.

5. Conclusions

A 3D finite element code for analysis of multiple cylindrical impacts on FRP laminated plate has been developed. The code is quite general in terms of number of impactors, time of impacts and location of impacts. Contact force histories, plate and impactor displacement, plate and impactor velocities have been studied for two successive cylindrical impacts and the subsequent delaminations at the interfaces due to such impacts have been assessed. Present work lead to the following important conclusions:

1. In case of multiple impacts, magnitude of contact force at various impact points depend upon the time interval between successive impacts in addition to mass and velocity of the impactors. Depending upon the relative velocities of the plate and the impactor at the time of contact magnitude of contact force will be different.

2. In case of multiple impacts, delaminations start from two distinct zones surrounding the impact points. However whether two delaminations will coalesce into one single delamination or remain two distinct delaminations depend upon the time interval of successive impacts in addition to the impactor velocity and the distance between the two impact points.

3. Delaminations at an interface near the impact points remain distinct for increasing time interval between the successive impacts. For quicker successive impacts, they coalesce into one big delamination.

References


