CREEP BEHAVIOR OF RC TENSILE ELEMENTS RETROFITTED BY FRP PLATES

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Abstract

Creep deformations in cracked FRP-reinforced RC tensile members are studied. Incremental compatibility and equilibrium equations are written. Creep laws for concrete and for steel-concrete and plate-concrete interfaces are defined making use of Bazant' solidification theory. The numerical solution of incremental problem is based on a one-dimensional finite difference scheme and solved via Newton-Raphson algorithm. Some numerical examples are presented. FRP-reinforcement is shown to be very effective to improve long-term behavior of RC elements by reducing crack width and deformability.

Introduction

The first theoretical and experimental studies on R/C beams retrofitted by FRP plates were devoted to the estimation of stress resultants causing failure of structural elements and to failure modes (e.g., ductile or brittle failures) [1, 2]. These aspects are in fact fundamental to establish safety margins and reliability of retrofit interventions, as well as to select the most effective materials to assure them.

Only very recently, researchers and engineers considered other very important aspects of the problem, like the behavior under service loadings [3 - 7] or the durability of retrofit interventions [8], [9]. Particularly important are creep behavior and evolution of delayed deformation with time, which may have significant consequences on the structural behavior. In the case of a retrofit intervention, for example, design errors and unreliable interventions with time can be done by ignoring delayed deformations (both of the structure to be retrofitted and of strengthening material). It is well recognized by researchers and producers that only with a complete knowledge of advantages and disadvantages of the use of these materials it is possible to extend these innovative techniques to many application fields in civil engineering. Nevertheless, studies devoted to time-dependent behavior of FRP reinforced RC beams are very few at the moment [10].

In the present study, the role of creep strains on the serviceability behavior of RC tensile members retrofitted by FRP-plates is studied. The structural model follows that proposed by the authors in [6, 7], where concrete member, steel bars and FRP plates are considered as different elements with bonding interfaces defined by appropriate bond-slip laws. For concrete creep, a constitutive model based on Bazant' solidification theory [11, 12] and recently improved by the authors [13] is adopted. Long-term behavior of bond slip for steel-concrete and plate-concrete interfaces is also considered.

The incremental governing equations are solved by means of a finite difference scheme. The formation of concrete tensile cracks is taken into account by imposing internal boundary conditions where concrete tensile stress is set to zero and continuity of steel and FRP plates is prescribed.

The model is used to estimate the long-term effectiveness of FRP strengthening of RC tensile members under service loadings. It is shown that retrofit with FRP plates reduces creep deformation and, consequently, the long-term elongation of R/C tensile member with respect to unreinforced case. In fact, FRP retrofit reduces significantly crack width increase with time, so preserving a significant value of concrete tension-stiffening in the cracked stage.

An Incremental Nonlinear Model for FRP-Reinforced Tensile Members

Kinematic and Equilibrium Relations

A R/C tensile member is strengthened by means of a couple of composite FRP plates glued along its external faces. A displacement-based model is adopted, originally proposed by the authors in [6, 7] and written here in incremental form.

The notation adopted for kinematic variables is reported in Figure 1. Subscripts c, s and p denote quantities related to concrete, steel bars and composite plates, respectively. Axial deformations of concrete, steel bars and FRP plates are denoted by ε_c, ε_s and ε_p.

Governing equations are written in incremental form, so that creep deformations with time may be included in the analysis. The general time increment may correspond to an external action (prescribed axial load or axial displacement) increment or to a constant action (classical creep or relaxation problems). Moreover, steel bars and carbon-fiber composite laminae are considered non viscous linear elastic materials.

According to [7], incremental equilibrium and compatibility equations may be written as:

$$\frac{\mathrm{d}\dot{u}_c}{\mathrm{d}x} = \frac{1}{E_c(t)\,A_c}\dot{N}_c\ , \qquad \frac{\mathrm{d}\dot{N}_c}{\mathrm{d}x} = c_s\,\dot{\tau}_s + 2b_p\,\dot{\tau}_p\,, \tag{1}$$

$$\frac{\mathrm{d}\dot{u}_s}{\mathrm{d}x} = \frac{1}{E_s\,A_s}\dot{N}_s\,, \qquad \frac{\mathrm{d}\dot{N}_s}{\mathrm{d}x} = -c_s\,\dot{\tau}_s\,, \tag{2}$$

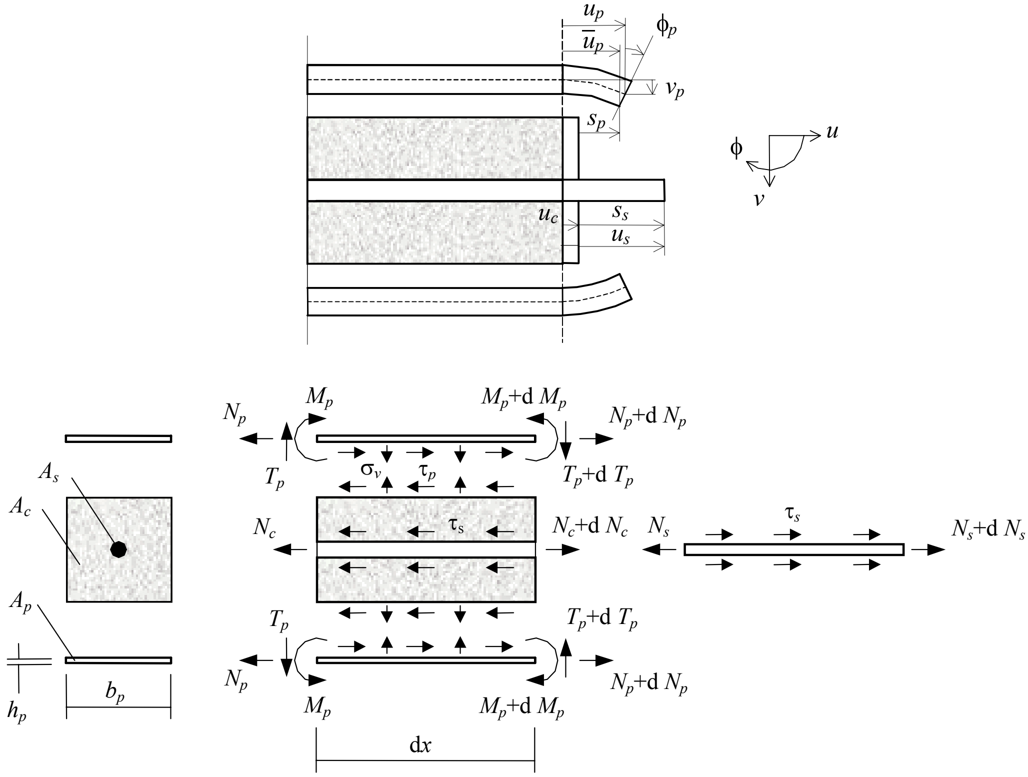


Figure 1: Notation adopted for displacements and forces for concrete, steel bars and plates.

$$\frac{\mathrm{d}\dot{u}_p}{\mathrm{d}x}=\frac{1}{E_p A_p}\dot{N}_p-\frac{h_p}{2E_p J_p}\dot{M}_p, \qquad \frac{\mathrm{d}\dot{v}_p}{\mathrm{d}x}=\dot{\varphi}_p, \qquad \frac{\mathrm{d}\dot{\varphi}_p}{\mathrm{d}x}=-\frac{1}{E_p J_p}\dot{M}_p, \tag{3}$$

$$\frac{\mathrm{d}\dot{N}_p}{\mathrm{d}x}=-b_p\,\dot{\tau}_p, \qquad \frac{\mathrm{d}\dot{T}_p}{\mathrm{d}x}=-b_p\,\dot{\sigma}_v, \qquad \frac{\mathrm{d}\dot{M}_p}{\mathrm{d}x}=\dot{T}_p-\dot{\tau}_p\frac{h_p}{2}b_p. \tag{4}$$

Dot denotes derivative with respect to time. Eqns (1), (2) give compatibility and equilibrium of concrete element and steel bars, respectively, with $c_s = n_b \pi \Phi$ (n_b, Φ = number and diameter of steel bars), A_c, A_s are areas of concrete and steel bars, $E_c(t)$, E_s are the corresponding elastic moduli and τ_s, τ_p denote bond shear stresses at the concrete-steel and concrete-plate interfaces (Figure 1). Eqns (3), (4) represent compatibility and equilibrium conditions for the FRP plates (with shear deformation neglected due to the small thickness), with u_p, v_p, ϕ_p, N_p, T_p, M_p denoting axial displacement, transverse displacement, rotation, axial force, shear and bending moment of the plates, respectively, with A_p, J_p, E_p being area, moment of inertia and Young modulus.

Constitutive Creep Equations

In the present model, the time dependent behavior of concrete is considered as linearly viscoelastic and described by means of a modified version of solidification theory, originally proposed by Bazant [11]. As for steel-concrete bond slip law, the relation for instantaneous loading adopted by CEB MC90 [14] is modified according to solidification theory in order to include the time dependent behavior. Finally, for the plate-concrete interface laws, viscous behavior of both adhesive and external cover of concrete is considered.

Creep Law for Concrete **-** *The Proposed Creep Law*

Solidification theory allows for the definition of an incremental constitutive law for ageing time-dependent materials. The basic assumption is that the material is considered as a varying composite whose components, a solidified (i.e. load carrying) material with volume fraction $v(t)$ and the complementary non solidified part with volume fraction 1- $v(t)$ are characterized by age-independent properties. Ageing of concrete with time is then governed by the adopted solidification law $v(t)$. The deposited layers of solidified constituent are assumed to be subject to the same total strain increment $\mathrm{d}\varepsilon$. Creep properties of solidified layers are defined through the micro-relaxation function $\Psi(t{-}t')$. Hence, the relation at time t between the (micro)stress $s[v(t)]$ (corresponding to a layer whose solidification started at $t'{=}\tau$) and total strain ε_c is given by:

$$s(\tau,t)=\int_\tau^t \Psi(t-t')\,\mathrm{d}\varepsilon_c(t'). \tag{5}$$

By superposition, using eqn (5) and global equilibrium condition for concrete, the relation between the macroscopic (applied) stress σ_c and the macroscopic strain ε_c can be written as:

$$\sigma_c(t)=\int_\tau^t \Psi(t-t')\,v(t')\,\mathrm{d}\varepsilon_c(t'). \tag{6}$$

In order to transform eqn (6) into an incremental form, a Dirichlet series expansion for the microscopic relaxation function is introduced:

$$\Psi(t-t')=\sum_{\mu=1}^{n} E_\mu\, e^{-(t-t')/\tau_\mu}, \tag{7}$$

where n is the number of terms adopted in the expansion (analogous to the number of units in a Maxwell chain, see Figure 2*a*), whereas E_μ , τ_μ (μ = 1,.., n) are the elastic moduli and the characteristic relaxation times of the units. Substituting eqn (7) into (6), straightforward algebra gives:

$$\sigma_c(t) = \sum_{\mu=1}^{n} \sigma_\mu(t)\,, \tag{8}$$

where the stress $\sigma_\mu(t)$, corresponding to the μth unit, is the solution of the differential equation:

$$\dot{\sigma}_\mu(t) + \frac{\sigma_\mu(t)}{\tau_\mu} = E_\mu \quad v_\mu(t) \quad \dot{\varepsilon}_c(t) \qquad (\mu=1,..., n)\,. \tag{9}$$

In the present model, two different solidification functions only are adopted. Function $v_1(t)$ governs ageing of first $n-1$ units, whereas $v_n(t)$ modulates ageing of the last unit, which is a degenerated purely elastic unit (i.e., $\tau_n \to \infty$), in order to ensure an asymptotic value of creep strain for $t \to \infty$, as prescribed by CEB MC90 [14]. The adopted solidification functions are:

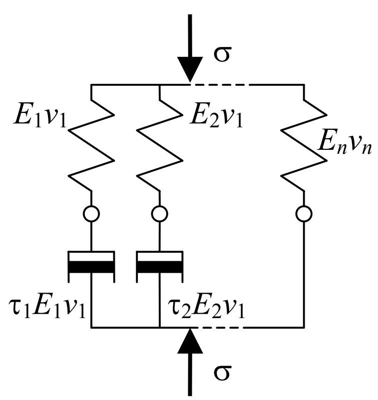
$$v_1(t) = \sqrt{e^{s_1(1-\sqrt{28/t})}}\,, \qquad v_n(t) = \sqrt{e^{s_n(1-\sqrt{28/t})}}\,, \tag{10}$$

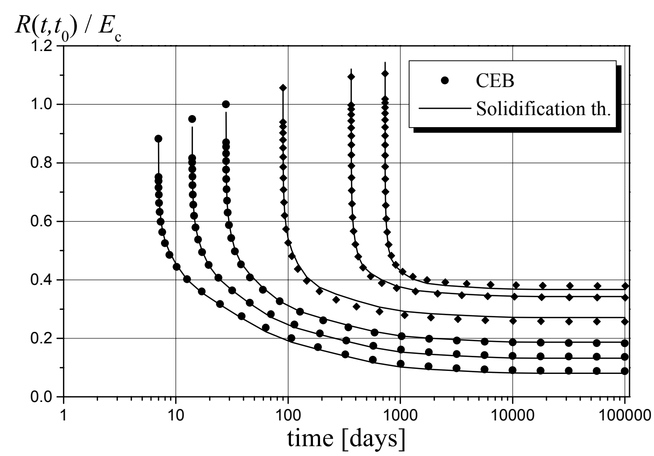
where time is expressed in days and s_1, s_n are free parameters. These expressions are analogous to the time evolution law of concrete Young modulus given by CEB MC90 [14].

For the numerical applications, 9 units are adopted, relaxation times are selected according to $\tau_\mu = 0.003 \cdot 10^\mu$ ($\mu = 1$,.., n-1), whereas parameters E_μ defining elastic moduli and s_1, s_N for solidification functions are obtained by best fitting procedure from CEB MC90 relaxation curves for concrete in compression (Figure 2*b*). These parameters are used here to predict creep in tension. Of course, this extension should be verified, even if it is normally adopted in the literature.

The Numerical Formulation

Incremental formulations are very useful in creep problems, since they do not require storage of the whole stress history (as required in the solution of convolution integrals), but the last value only of state variables must be stored before the subsequent (finite) time step. Nevertheless, classical constant step integration methods cannot be adopted to obtain results over the entire time interval of practical interest since, using small time steps (required at the beginning of loading) causes a great computational





(*a*) (*b*)

Figure 2: (*a*) Maxwell chain composed of N units; (*b*) best fitting of CEB relaxation curves using solidification theory (RH%=50, f_{cm}=38 Mpa, 2A$_c$/u=50 mm).

effort to predict the behavior after several months or years.

In the present study, the exponential algorithm is adopted, originally proposed by Zienkiewicz [15] and Bazant [16, 17] for linear viscoelasticity and later extended to non linear problems [18, 19]. It is based on the exact integration of differential equation (9) over the time step $\Delta t_r = t_r - t_{r-1}$ ($r = 1,\dots, N$), where $t_0, t_1, \dots, t_N$ are the time instants where solution must be evaluated. By multiplying both sides of eqn (9) by the integration function:

$$f_\mu(t) = e^{\int_{t_{r-1}}^{t} \frac{dt'}{\tau_\mu}} \tag{11}$$

and performing integration over the time step Δt_r yield:

$$\sigma_\mu(t_r) = e^{-f_\mu(t_r)}\left[\sigma_{\mu,r-1} + \int_{t_{r-1}}^{t_r} E_\mu v_\mu(t')\, e^{-f_\mu(t')} d\varepsilon(t')\right], \tag{12}$$

where $\sigma_{\mu,r-1}$ is the value of stress of μth Maxwell unit at the previous (t_{r-1}) time instant. If Δt_r is sufficiently small or the variation of the ageing function $v_\mu(t)$ over the time step is small enough, then the r.h.s. of eqn (12) can be integrated analitically, so obtaining the stress at the t_r time instant:

$$\sigma_{\mu,r} = \sigma_{\mu,r-1}\, e^{-\frac{\Delta t_r}{\tau_\mu}} + E_\mu\, v_{\mu,r-1/2}\, \lambda_{\mu,r}\, \Delta\varepsilon_r \tag{13}$$

where:

$$\lambda_{\mu,r} = (1 - e^{-\frac{\Delta t_r}{\tau_\mu}})\frac{\tau_\mu}{\Delta t_r}, \qquad v_{\mu,r-1/2} = \tfrac{1}{2}\left(v_{\mu,r-1} + v_{\mu,r}\right). \tag{14}$$

Substituting eqn (13) into (8) gives the following incremental pseudo-elastic stress-strain relation at t_r:

$$\Delta\varepsilon_r = \frac{1}{E_r''}\Delta\sigma_r + \Delta\varepsilon_r'', \tag{15}$$

where:

$$E_r'' = \sum_{\mu=1}^{N} \lambda_{\mu,r} E_\mu v_{\mu,r-1/2} \qquad \Delta\varepsilon_r'' = \frac{1}{E_r''}\sum_{\mu=1}^{N}\left(1 - e^{-\frac{\Delta t_r}{\tau_\mu}}\right)\sigma_{\mu,r-1}. \tag{16}$$

In the numerical procedure, time steps are chosen equally spaced in the log-scale, so that few time steps are required to cover the whole period of interest (years). Convergence properties of exponential algorithm for non linear problems are discussed in [19], with reference to concrete damage in compression.

Eqn (15) can be easily used to compute the strain increment during the time step Δt_r, since pseudo-elastic modulus E_r'' and pseudo-deformation $\Delta\varepsilon_r''$ depend on quantities known after the previous time step ($r-1$) or function of time.

Bond-Slip Steel-Concrete Creep Law

It is well recognized that creep deformations can affect also the behavior of steel-concrete interface; the main effect of creep deformation is the increasing with time of slip and of plate anchorage length close to discontinuities (plate ends or concrete cracks). The experimental results presented by

Sato et al [20] show the time-dependent changes of bond stresses, steel strains and concrete strains along the longitudinal axis of RC members in tension.

In the present study, solidification theory is also used to define a time-dependent bond-slip steel-concrete law. Five units are adopted, where the last one is a degenerated unit, so that $\tau_5 \to \infty$ and $\lambda_{5,r} \to 1$. If, as a first stage, a linear shear stress-slip law is adopted, the integration of governing relation according to exponential algorithm gives the following incremental law:

$$\Delta\tau_{s,r} = K_s''(t)\left[\Delta s_{s,r} - \Delta s_{s,r}''\right], \tag{17}$$

where $s_s = u_s - u_c$ is the steel-concrete slip, i.e. the relative displacement of two points belonging to steel and concrete which were initially in contact, τ_s is the corresponding interfacial shear stress, and:

$$K_s''(t) = K_0 \cdot \left(\sum_{\mu=1}^{4} \lambda_{\mu,r}\, k_\mu\, \nu_{\mu,r-1/2} + k_5\, \nu_{5,r-1/2} \right), \tag{18}$$

where K_0 is the instantaneous stiffness for t_0=28 days and k_1,… k_5 are non dimensional coefficients.

Eqns (17), (18) can be obtained following a procedure analogous to that described in the previous Section. Two solidification functions v_1, v_5 are adopted, selected in the same form of eqns (10). Moreover, stiffness constants $K_1 \dots K_5$ are defined following the same procedure adopted for concrete Maxwell units and using, for best fitting, the same relaxation curves. The residual term $\Delta s_{s,r}''$ in incremental eqn (17) is defined, in analogy to eqn (16), as:

$$\Delta s_r'' = \frac{1}{K_{s,r}''} \sum_{\mu=1}^{4} \left(1 - e^{-\Delta t_r / \tau_\mu} \right) \tau_{s\mu,r-1} \,. \tag{19}$$

The creep law (17) is then extended to cover non linear bond-slip behavior, as typically assumed for instantaneous loadings. Non linearity is introduced as a modification of elastic stiffness K_0, which is substituted by a tangent stiffness K_{tg} depending on slip level s_s. Starting from classical non linear bond slip relation proposed by CEB-MC90 model [14]:

$$\tau_s = \left(\frac{\tau_{s,\max}}{s_{s,\max}^{\alpha}} \right) s^{\alpha} \tag{20}$$

where, typically, $\tau_{s,\max} = 1.5 \cdot f_{ck}^{1/2}$, $s_{s,max} = 0.6$ mm and $\alpha = 0.4$, eqn (20) is rewritten in incremental form as:

$$\Delta\tau_s = K_{tg}(s_s)\,\Delta s_s\,, \qquad \text{where} \qquad K_{tg}(s_s) = \left(\frac{\tau_{s,\max}}{s_{s,\max}^{\alpha}} \right) \alpha\, s_s^{\alpha-1}\,. \tag{21}$$

Setting $f_{NL}(s_s) = K_{tg}(s_s) / K_0$, incremental bond-slip eqn (17) is extended to non linear range as:

$$\Delta\tau_{s,r} = f_{NL}(s_{s,r})\, K_s''(t)\left[\Delta s_{s,r} - \Delta s_{s,r}''\right]. \tag{22}$$

Assumption adopted to obtain eqn (22) is analogous to that currently used for non linear creep modeling of concrete, i.e. non linearity is taken into account as a non linear function multiplying linear creep strain [21]. This assumption is considered valid for medium stress levels, whereas for higher stresses the non linear creep mechanism is more complex [22]. Analogous assumption is introduced by

CEB MC90 [23 - 25], even if a different creep function is introduced, i.e. $k_t = (1+10\,t)^a - 1$, where t is the load duration (in hours) and coefficient a depends on concrete quality (a = 0.08 for normal strength concrete).

FRP plate-Concrete Creep law

The interface plate-concrete creep law must take into account shear deformation of both adhesive and external concrete cover. Numerical and experimental tests showed that a 30÷50 mm depth cover participates to interface shear compliance [26, 7]. Only taking that contribution into account, the actual decay length of end effects (about 80÷100 mm) can be explained, whereas 10 mm length is obtained by considering adhesive compliance only.

In the present study, solidification theory is used to define shear compliances of both adhesive and concrete, leading to equations analogous to those obtained in previous Sections:

$$\Delta s_{A,r} = \frac{1}{K''_A(t)}\Delta\tau_{p,r} + \Delta s''_{A,r}\,, \qquad \Delta s_{C,r} = \frac{1}{K''_C(t)}\Delta\tau_{p,r} + \Delta s''_{C,r}\,, \tag{23}$$

where subscripts A and C stand for terms related to adhesive and concrete, respectively, whereas τ_p is the plate-concrete shear stress. Total plate-concrete slip increment $\Delta s_p = \Delta s_A + \Delta s_C$ is then obtained in the form:

$$\Delta s_{p,r} = \left(\frac{1}{K''_A(t)} + \frac{1}{K''_C(t)}\right)\Delta\tau_{p,r} + \Delta s''_{p,r}\,, \tag{24}$$

where $\Delta s''_{p,r} = \Delta s''_{A,r} + \Delta s''_{C,r}$. According to eqn (24), different creep formulations for adhesive and concrete can be adopted. For instance, in the present work, non ageing two-term creep formulation is introduced for adhesive whereas, for concrete compliance contribution, an expression similar to that adopted for concrete is used.

Finally, plate-concrete slip is written in terms of unknown displacements as (see Figure 1):

$$s_p = \bar{u}_p - u_c = u_p - \varphi_p \frac{h_p}{2} - u_c\,, \tag{25}$$

where $\bar{u}_p$ is the displacement of the surface of the glued plate and h_p is the plate thickness.

Numerical Solution of Incremental Equations via Finite Difference Method

Substitution of constitutive eqns (15), (22), (24) into (1)-(4) leads to a system of incremental equations for the rth time increment, which is rewritten in the compact form:

$$\frac{\mathrm{d}\Delta\mathbf{y}_r(x,t)}{\mathrm{d}x} = \mathbf{A}_{tg}(\mathbf{y}_r, x, t)\;\;\Delta\mathbf{y}_r(x,t) + \Delta\alpha_r''(x,t) \qquad 0 \le x \le L\,, \tag{26}$$

where $\Delta\mathbf{y}_r$ is the vector collecting the increments at the rth step of unknown function $\mathbf{y}$:

$$\mathbf{y}^T = \left\{N_s, N_p, N_c, M_p, T_p, u_s, u_p, u_c, v_p, \varphi_p\right\} \tag{27}$$

and $\Delta\alpha_r''(x,t)$ is the vector, known because only related to the solution at the previous (r–1)th step, collecting terms due to creep deformation (see eqns (15), (22), (24)).

Moreover, the boundary conditions to be prescribed at the two ends $x = 0$, L of the tensile member can be written as:

$$\mathbf{B}_a \Delta\mathbf{y}_r(0) + \mathbf{B}_b \Delta\mathbf{y}_r(L) = \Delta\alpha_r \,. \tag{28}$$

Vector $\Delta\alpha_r$ takes different forms if axial force increment ΔN or, alternatively, axial elongation ΔL is prescribed, in order to simulate creep or relaxation tests, respectively.

Incremental boundary value problem (27), (28) is solved here via finite difference method, by writing spatial derivatives (along x), i.e. the l.h.s. of eqn (26), by finite difference approximation (see [7] for details). The interval [0, L] is divided using J nodes at uniform distance h, i.e., $0=x_0<x_1<\ldots<x_J=L$. For the general j-th interior mesh point ($0<j<J$), the derivative in Eqn (26) is replaced by the central finite difference approximation $(\Delta\mathbf{y}_{j+1}-\Delta\mathbf{y}_j)/h$ centered at $x_{j+1/2}$. Finite difference equations, together with boundary conditions (28), lead to the non-linear system:

$$\begin{vmatrix} \mathbf{S}_1 & \mathbf{R}_1 & & & & & \\ & \ldots & \ldots & & & & \\ & & \mathbf{S}_j & \mathbf{R}_j & & & \\ & & & \ldots & \ldots & & \\ & & & & \mathbf{S}_J & \mathbf{R}_J \\ \mathbf{B}_a & & & & & \mathbf{B}_b \end{vmatrix} \begin{vmatrix} \Delta\mathbf{y}_{1,r} \\ \Delta\mathbf{y}_{2,r} \\ \ldots \\ \ldots \\ \Delta\mathbf{y}_{J-1,r} \\ \Delta\mathbf{y}_{J,r} \end{vmatrix} = \begin{vmatrix} \mathbf{0} \\ \mathbf{0} \\ \ldots \\ \\ \ldots \\ \Delta\boldsymbol{\alpha}_r \end{vmatrix} + \begin{vmatrix} \Delta\boldsymbol{\alpha}''_{1,r} \\ \Delta\boldsymbol{\alpha}''_{2,r} \\ \ldots \\ \ldots \\ \Delta\boldsymbol{\alpha}''_{J-1,r} \\ \Delta\boldsymbol{\alpha}''_{J,r} \end{vmatrix} \tag{29}$$

where:

$$\mathbf{S}_j = -\frac{1}{h}\mathbf{I} - \frac{1}{2}\mathbf{A}_{tg}(\mathbf{y}_{j,r},\ x_j, t)\,,\ \mathbf{R}_j = -\frac{1}{h}\mathbf{I} + \frac{1}{2}\mathbf{A}_{tg}(\mathbf{y}_{j+1,r},\ x_{j+1}, t) \qquad j = 1, 2, \ldots, J \tag{30}$$

and $\mathbf{I}$ is the identity matrix. The second term at the r.h.s. of Eqn (29) contains the prescribed (at rth step) values related to creep deformations, as obtained from solidification theory.

To solve the incremental non linear system (29), for a given increment of applied action (axial force N or elongation ΔL), the solution is obtained iteratively by using modified Newton-Raphson method. According to eqn (29), the simplest forward incremental formulation has been adopted. To obtain an accurate solution, small step must be used, in order to avoid error accumulation. The step size has been calibrated, with reference to instantaneous loading, by comparison with a solution obtained by a non incremental Newton-Raphson solution procedure, i.e. by adopting total values of unknown variables and secant matrix (see [7]). In order to adopt larger increments, for the creep problem where the adoption of an incremental formulation is mandatory, improved solution procedures are under study.

When tensile stress in concrete reaches the strength f_{ct} in a node, say the jth node, the formation of a transverse crack in concrete is imposed by setting a vanishing value of tensile stress. Crack produces relative displacement in concrete $w=u_{c,j+1}-u_{c,j}$ between two subsequent nodes, while continuity is preserved for steel and FRP-plates. This condition is introduced by replacing the jth row of system in eqn (29) by an internal boundary condition, imposing the modified equilibrium and compatibility conditions. This equation can be written in the form [7]:

$$\mathbf{B}_c \Delta\mathbf{y}_{r,j} + \mathbf{B}_d \Delta\mathbf{y}_{r,j+1} = \mathbf{0}. \tag{31}$$

Numerical Examples

The proposed model has been used for the study of long-term behavior of RC tensile members retrofitted by external FRP carbon fibre plates. Mechanical and geometrical properties are reported in Figure 3. Two different plate thicknesses (h_p=0.33 mm and 1.016 mm) and an unplated member have been considered.

Creep tests have been simulated, where a loading rate of 5000 kN/day has been prescribed until the maximum load is reached, followed by a long period of sustained constant load (about 30 years). Two different cases of maximum loading have been considered, a low level load where no concrete cracking occurs, and a higher load level. Figures 4a, 4c, 4e show the corresponding axial loading – axial elongation curves, compared with those obtained for instantaneous loading (no creep effects). Creep long term elongation is smaller for the low reinforced case. Moreover, Figures 4b, 4d, 4f depict the time-variation of axial strain in concrete at x=840 mm, i.e. far from sections where concrete crack occurs. Figures show deformation during both loading increase (before 0.021 days) and sustained loading time interval (from 0.021 to 10^4 days). Unplated member shows larger deformation with respect to reinforced cases; for instance, for 93 kN sustained loading, strain in concrete is 19 percent higher with respect to 0.33 mm - plated member and 26 percent higher with respect to 1.016 mm -plated case.
Finally, for the h_p=1.016 mm case, the profile along the member of most important stress and displacement variables are reported in Figures 5a-f. Dashed line stands for quantities at the end of loading (before the first crack formation), solid line after crack formation, and thick solid line after the period of sustained loading. Due to crack formation, stress in plate increases about seven time with respect to (uncracked) state I solution. Hence, plates give a high contribution to axial stiffness in the cracked range. Moreover, due to significant concrete creep strain, axial stress in both steel bars and plates increases significantly with time. Finally, it is worth noting that steel-concrete slip s_s and transmission length of end effects increase with time, whereas maximum values of corresponding shear stresses decrease significantly.

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$L \times b \times h$ (mm)	800 × 150 × 150
E_c (N/mm^2)	33640
F_c (N/mm^2)	36.4
F_{ct} (N/mm^2)	3.64
A_s (mm^2)	3.14
E_s (N/mm^2)	200000
$b_p \times h_p$ (mm)	150 × 1.016 (or 0.33)
E_p (N/mm^2)	230000
H_g (mm)	1.5
E_g (N/mm^2)	1212.9

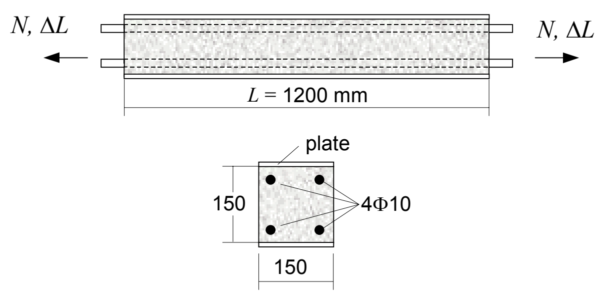

Figure 3: Geometry and mechanical properties of FRP-reinforced tensile member considered in numerical examples.

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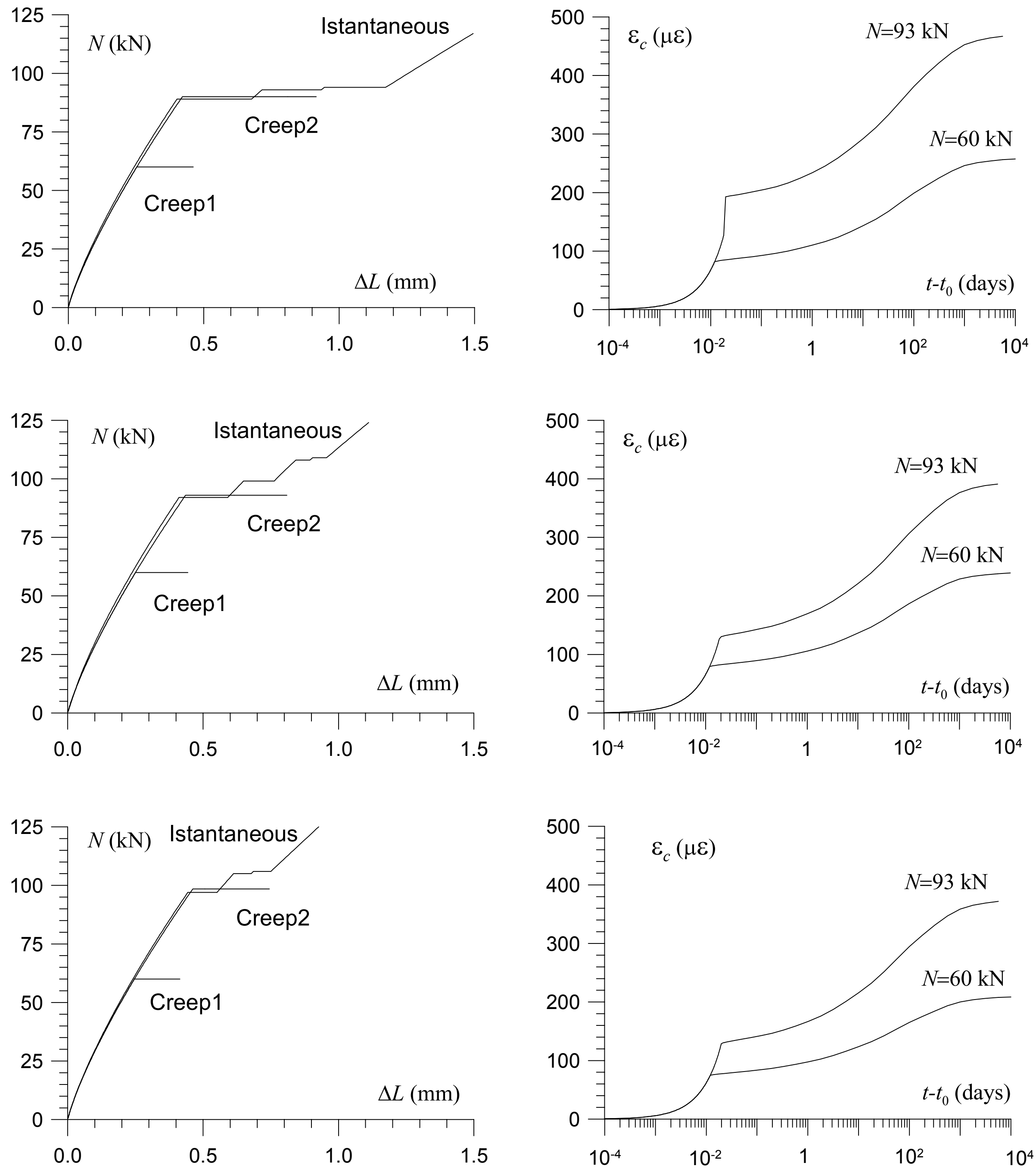

Figure 4: Instantaneous and long term force-elongation diagrams and strain evolution with time. (a ,b) unplated, (c, d) h_p=0.33 mm; (e, f) h_p=1.016 mm.

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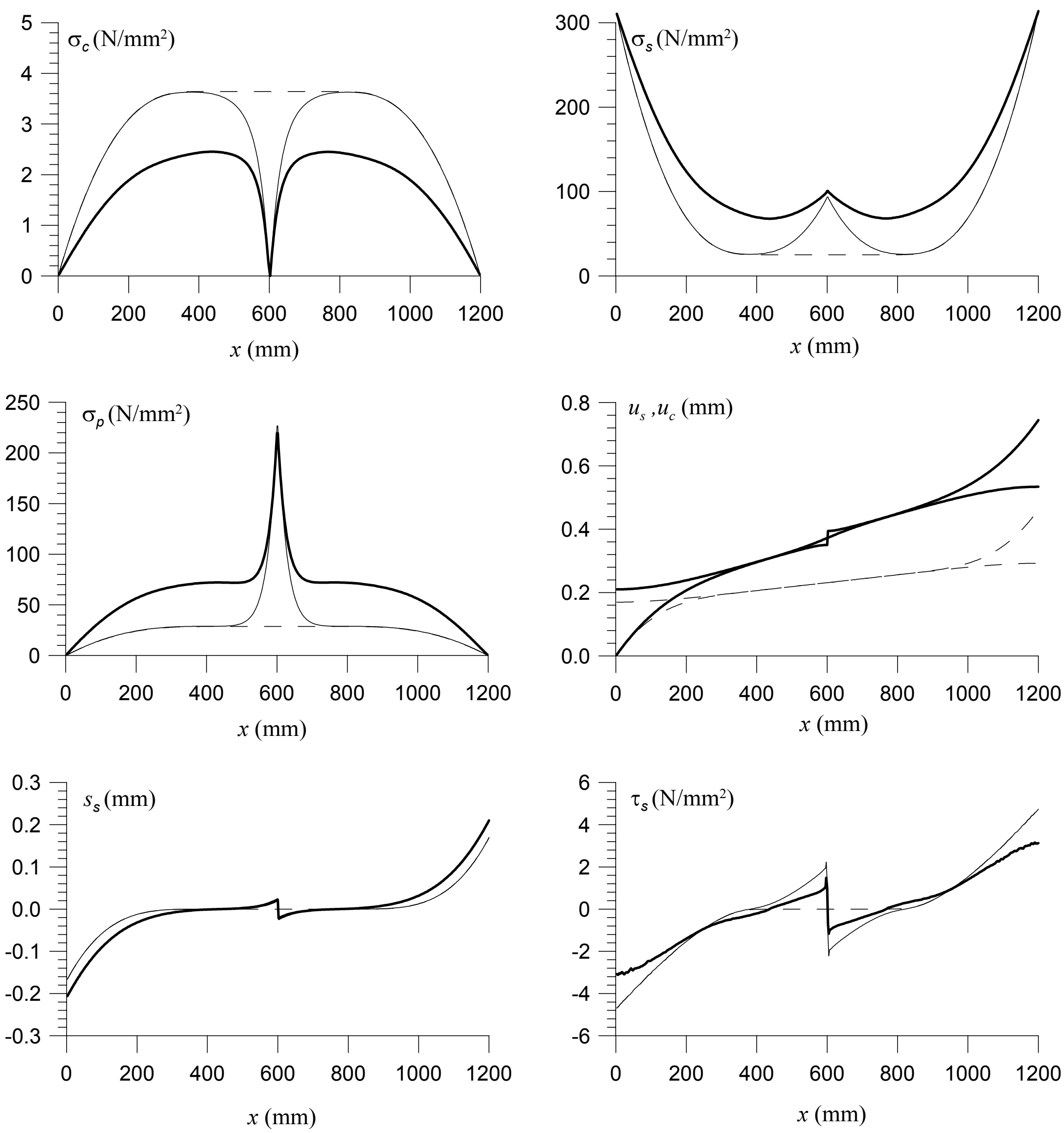

Figure 5: FRP-reinforced tensile member (h_p=1.016 mm): (*a*) concrete stress; (*b*) steel stress; (*c*) stress in FRP plates; (*d*) Steel and concrete displacements; (*e*) steel-concrete slip; (*f*) steel-concrete bond shear (---: before concrete crack; —: after crack; —: long time values: 30 years).

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