

# ANALYTICAL MODELING ON DEBONDING FAILURE OF FRP-STRENGTHENED RC FLEXURAL STRUCTURES

*Dr. Hedong Niu, Ibaraki University, Hitachi, Japan*  
*Prof. Zhishen Wu, Ibaraki University, Hitachi, Japan*

## Abstract

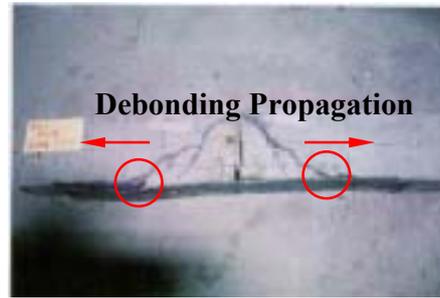
Effective application of FRP to concrete structure is not possible until a fundamental understanding of the mechanics and failure mechanisms of the retrofit system is available. This paper is mainly focused on developing a methodology for predicting the debonding failure caused by intermediate flexural cracks in FRP-strengthened R/C beam. Firstly, based on the strain compatibility, an iterative analytical method is presented to predict the structural response of strengthened beam. Then using concept of fracture mechanics, a comprehensive study is directed to clarifying debonding failure mechanisms corresponding to experimental results. Finally, a unified energy-based model is established to predict the debonding failure in FRP-strengthened R/C beam.

## Introduction

In the past twenty years, fiber reinforced plastic (FRP) laminates widely and successfully applied in aerospace structures have gradually been accepted as a highly attractive alternative in lieu of their steel counterpart in the field of strengthening and upgrading of civil engineering structures. This can be attributed to their superior characteristics such as light weight, high tensile strength, corrosion resistance, good tailorability and ease of application. These FRP materials can be externally bonded to the tension face of concrete structures with any desirable shape via a thin layer of epoxy adhesive and thus enhance stiffness and strength of the structures to be strengthened. As we know, the pivot to this bond technique is to ensure perfect composite action between FRP laminates, failure of which may invalidate the stress transfer from concrete substrate to FRP reinforcement and cause undesirable premature failure prior to the theoretically expected load. As shown in **Figure 1**, FRP-strengthened concrete beams may be subjected to unfavorable failure modes in addition to crushing of concrete and shear failure, such as rupture of FRP laminates, delamination of FRP initiated from the cut-off point, peeling-off of FRP caused by shear crack and debonding of FRP caused by intermediate flexural crack. Among them, crushing of concrete, shear failure and rupture of FRP can be avoided in the structural design and their corresponding ultimate strength of structural member can be predicted using conventional RC beam theory. So far considerable research has been directed to investigating the phenomenon of interfacial shear and normal stress concentrations at the cut-off point of FRP and the corresponding failure criteria have been developed for predicting the delamination of FRP initiated from the FRP ends (e.g. [1-5]). Different from other researchers, Zhang et al. [6] suggested a plausible mode of failure which is controlled by the characteristics of the individual teeth in between adjacent cracks in the concrete cover and proposed a theoretical model for predicting the premature plate peeling failure load which depends on the size of stabilized crack spacings. The peeling-off of FRP initiated from shear cracks of concrete can be prevented by a rational design and much research work on FRP shear strengthening has been done [7-9]. However, very limited literatures can be found concerning the debonding of FRP caused by intermediate flexural cracks generally located near the maximum moment region, which is considered to be a more dominant failure mode than the delamination of FRP at curtailment zone for the strengthened

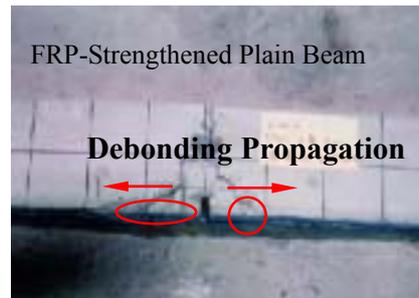
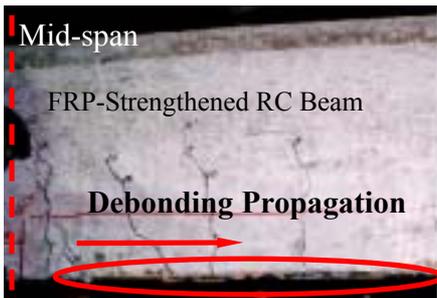


(a) Rupture of FRP



(b) Delamination of FRP at FRP ends

(c) Peeling-off of FRP by Shear Cracks



(d) Debonding of FRP Caused by Intermediate Flexural Cracks

**Figure 1.** Failure Modes Observed in FRP-Strengthened Concrete Beams

beam with thinner FRP sheets. A thorough understanding of such kind of debonding failure mechanism is crucial to developing the corresponding prediction method for civil engineers.

Recent studies [10-12] have demonstrated that proper understanding and modeling of FRP-concrete interface-related phenomena and failures could be improved via the application of fracture mechanics theories. The traditional strength-based theory can only predict the local fracture but not the ultimate failure load of the retrofitted structures. Wu and Niu [13-14] and Niu and Wu [15] theoretically investigated the effect of flexural cracks on interfacial stress distribution and pointed out that it is reasonable to use fracture mechanics concept for prediction of debonding failure load. Recently, Niu and Wu [16] clarified the debonding behavior and failure mechanism due to multiple flexural cracks in FRP-strengthened R/C beams through performing nonlinear fracture mechanics-based finite element analysis. It is shown that debonding behavior and the ultimate load of debonding failure are significantly influenced by whether crack spacing is less than the effective transfer length of FRP sheets or not and interfacial fracture energy.

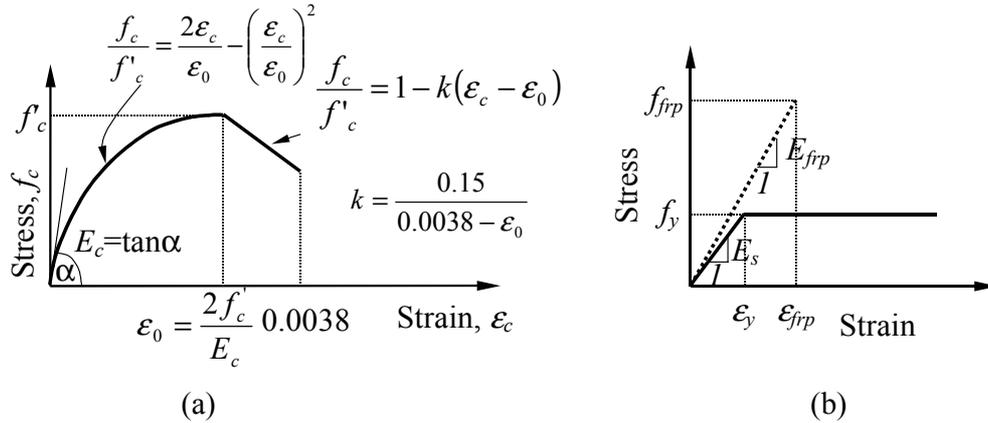
In this paper, particular emphasis is placed on developing an analytical model for predicting the debonding failure load caused by flexural cracks, which consists of section analysis based on strain compatibility, concept of fracture mechanics and simplified assumptions based on debonding mechanisms. Firstly, an iterative analytical method to analyze the structural response of FRP-strengthened RC beam is established based on strain compatibility and equilibrium. Then debonding mechanisms are interpreted for different crack patterns observed in practical experiments and the corresponding analytical prediction model is proposed. Finally, some discussions are made on model parameters.

## Structural Response in FRP-strengthened RC Beams

A doubly reinforced rectangular section is illustrated in this section to develop an iterative analytical procedure to predict the structural response to load application. This analytical study is based on the strain compatibility, equilibrium, and choice of material constitutive relations for concrete, reinforcing steel and FRP.

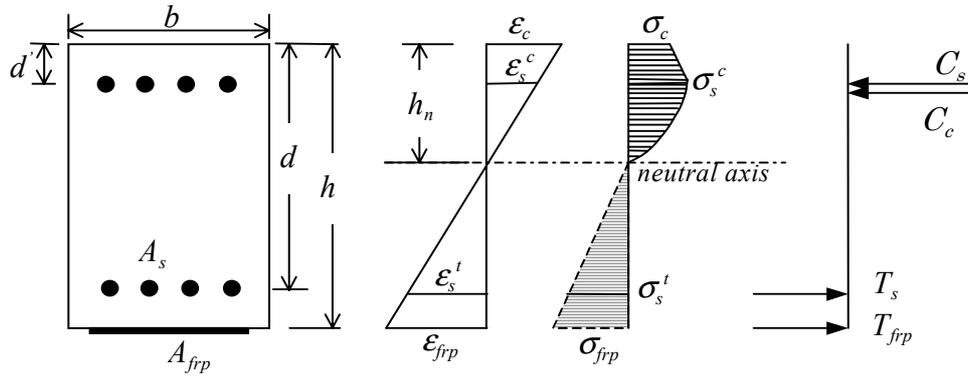
In the analysis, the following assumption are made:

- Linear strain distribution throughout the full depth of the section;
- No slip between the longitudinal reinforcing steel and the surrounding concrete;
- No slip between the external FRP reinforcement and the concrete substrate;
- No premature FRP separation or shear failure is accounted for;
- The tensile strength of the adhesive is ignored;
- No tensile strength is considered after cracking.

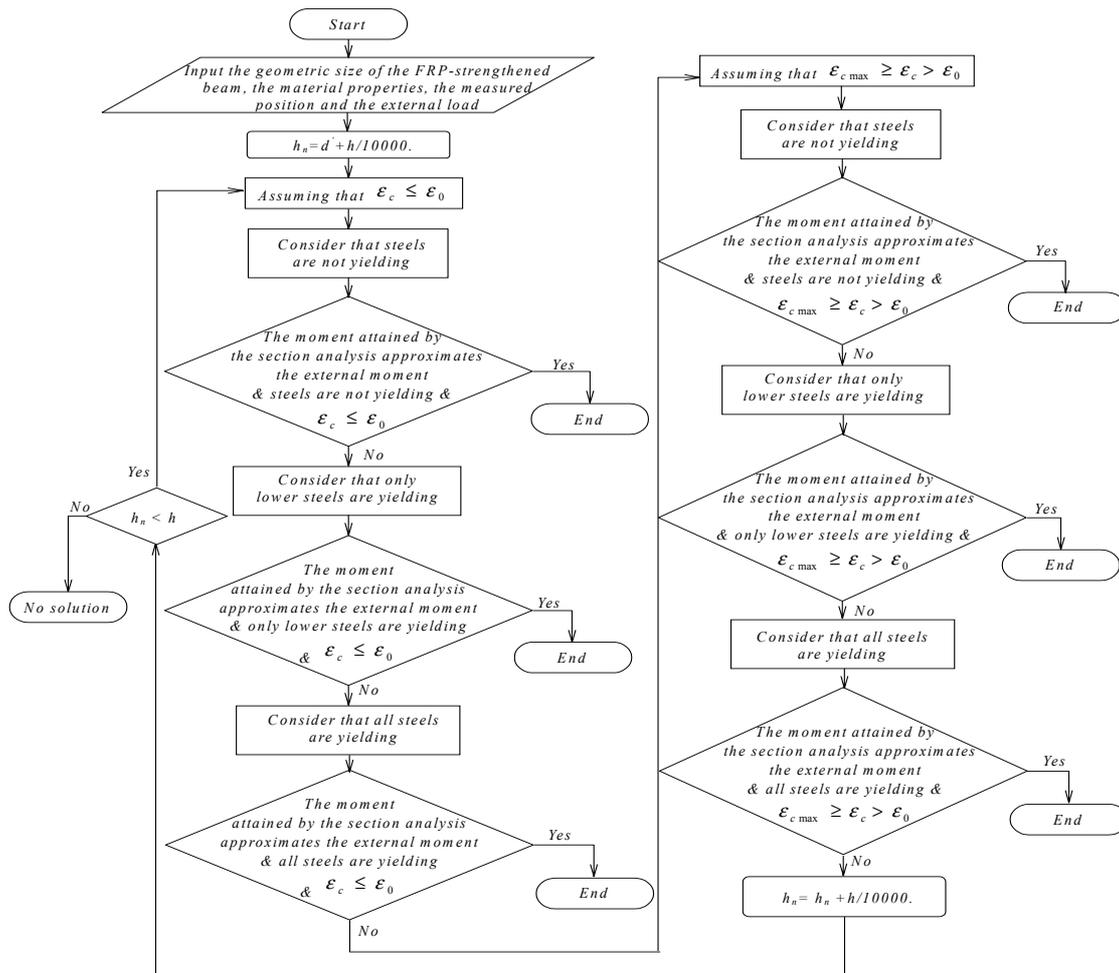


**Figure 2.** Constitutive Relationships: (a) Idealized Stress-Strain Curve for Concrete in Uniaxial Compression (after [17]); (b) Reinforcing Steel and FRP

To provide highly accurate prediction, concrete is assumed to follow the widely-used stress-strain curve proposed by Hognestad [17], reinforcing steel is modelled by elastic perfectly plastic curve in tension and compression, and FRP materials are assumed to behave linear elastically until to failure, as shown in **Figure 2**, where  $f'_c$ =compressive strength of concrete;  $f_c$ =stress in concrete;  $\epsilon_0$ = strain in concrete corresponding to compressive strength;  $\epsilon_c$ = strain in concrete;  $E_c$ =initial elastic modulus of concrete;  $f_y$ =yield strength;  $E_s$ =elastic modulus of steel;  $\epsilon_y$ = yield strain in reinforcing steel;  $f_{frp}$ =tensile strength of FRP;  $E_{frp}$ =elastic modulus of FRP;  $\epsilon_{frp}$ = ultimate tensile strain in FRP.



**Figure 3.** Strain, Stress and Force Distribution at Section



**Figure 4.** Flow Chart for Calculating FRP Stress for a Given Load

Based on the strain compatibility and equilibrium of internal forces, FRP stress or external load can be predicted for a specific loading stage. **Figure 3** shows the strain, stress and force distribution along the depth of cross-section. In view of nonlinear behavior of concrete and reinforcing steel, this

analysis should be performed by an iterative procedure. Every possible case should be checked for a given load or FRP stress at a certain section, such as whether the strain in the extreme fiber of concrete in compression is larger than  $\epsilon_0$  or not, compression steel or tension steel yields or not. In the analysis, the ultimate compressive strain of concrete  $\epsilon_{cmax}$  is assumed to be 0.0035.

With respect to calculation of external load for a given FRP stress, it can be very easily performed as follows: firstly assume  $\epsilon_c < \epsilon_0$ , calculate the neutral axis  $h_n$  and check whether the assumed conditions is met, if not, then assume  $\epsilon_c < \epsilon_0$  and repeat the same procedure until the calculation result agrees with the assumption condition; secondly, the location of the resultant of compressive force can be easily determined on the basis of the calculated neutral axis; and finally, the external load can be determined by the equilibrium of the moment at given section.

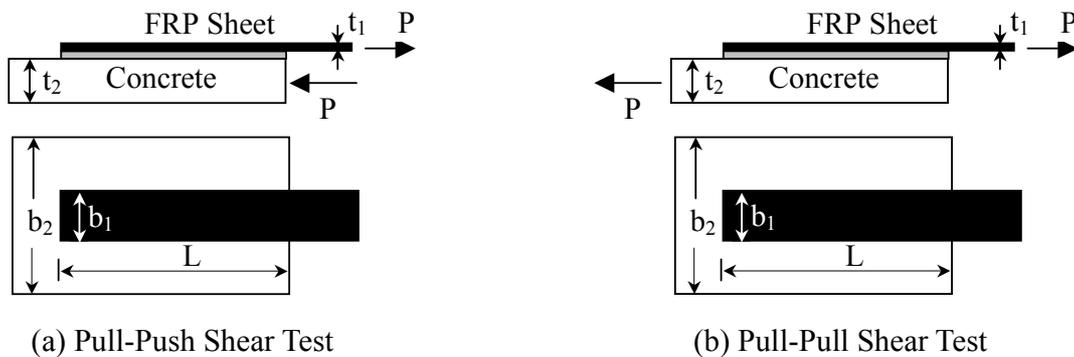
Determination of FRP stress is an inverse operation to the above stated and uses an iterative procedure following **Figure 4**.

Detailed analytical equations from the equilibrium of internal force can be referred to [14] and it was found that either external load or FRP stress can be predicted with high accuracy by compared to experimental results [18]. This kind of section analysis based on strain compatibility can be used to predict the crushing of concrete and rupture of FRP. It is evident that such analysis can hardly provide a reasonable prediction for debonding failure caused by intermediate flexural crack, which can be attributed to the fact that debonded section violates the assumption used in section analysis and knowledge about such debonding mechanisms is still lacking.

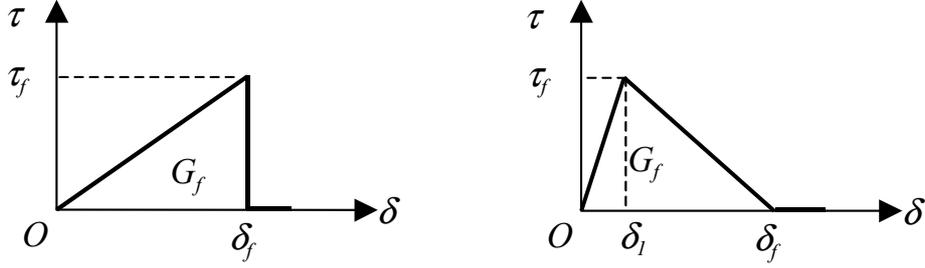
### Bonding and Debonding Mechanisms

As shown in **Figure 1d**, debonding of FRP laminates is often observed to be initiated from the ends of flexural cracks near the maximum moment region, with subsequent propagation out to the ends of FRP. Such debonding follows two possible paths: along the interface between adhesive layer and concrete substrate or through the concrete substrate adjacent to the bond interface. Generally, the latter mode is encountered in FRP-strengthened RC beam provided with good bond condition. Despite where debonding occurs, it can be regarded that debonding propagation resembles mode II fracture more closely than mode I fracture because FRP laminates are primarily loaded in tension and the adhesive is primarily in shear providing the necessary shear connection between concrete and FRP. In practice, such debonding propagation may be associated with mode I fracture within concrete surface layer [19], but the integrated effect can be similar to mode II fracture behavior, which lies in the fact that only a thin layer of concrete adheres to the FRP debonding surface.

#### Simple Shear Test



**Figure 5.** Simple Shear Test on FRP-Bonded Concrete Prism



(a) Linear Bond-Slip Curve

(b) Bilinear Bond-Slip Curve

**Figure 6.** Interfacial Constitutive Relationships

To investigate the bonding and debonding mechanisms for FRP-bonded concrete structures, considerable research has been conducted on pure shear test on FRP-bonded concrete prisms shown in **Figure 5**. It is worthy of being noted that Täljsten [20] firstly presented the use of fracture mechanics approaches for the plate bonding technique. The derived formula was solved analytically for simplified bond-slip curve without softening behavior. However, the author pointed that nonlinear equation derived for a realistic bond-slip curve could only be used for numerical calculations. Recently, Yuan et al. [21] and Wu et al. [22] introduced several interfacial constitutive laws (as shown in **Figure 6**) describing pre- and post- cracking behavior of FRP-concrete interface to analytically solve the nonlinear shear transfer problems of pull-push and pull-pull shear tests. It shows that fracture mechanics can be used to well explain the debonding initiation, propagation and final failure, which shames the strength theory-based method. Moreover, it provides a simple expression for determining the load-carrying capacity.

Provided that the bond length  $L$  is larger than the effective transfer length, the maximum transferable load in pull-push or pull-pull shear test can be expressed in the same form with respect to the choice of different interfacial constitutive relationships:

$$P_{\max} = b_l \sqrt{2G_f E_1 t_1} \quad (1)$$

where  $E_1$ ,  $t_1$  and  $b_l$  are elastic modulus, thickness and width of FRP, respectively;  $G_f$  is the interfacial fracture energy consumed for debonding failure.

### **FRP-Strengthened R/C Beams**

As for FRP-strengthened R/C beam, generally intermediate flexural crack initiated debonding is observed to be accompanied with two different crack patterns: localized crack pattern more often than not encountered in plain concrete beam strengthened with FRP laminates and distributed crack pattern in strengthened RC beam, which is shown in **Figure 1d**. Due to the effect of moment and different crack patterns, it may not be taken for granted that the debonding mechanisms are same for strengthened beam and FRP-bonded concrete prism and the expressions derived from simple shear test can be kept intact to apply to strengthened beam. To this end, the authors [13][15][16] conducted a comprehensive study on stress transfer, debonding propagation and failure mechanism using theoretical and numerical method. In what follows, the similarities and dissimilarities are addressed between debonding mechanisms in cracked FRP-strengthened R/C beam and simple shear test.

Based on theory of elasticity, Wu and Niu [13] theoretically investigated the effect of flexural cracks around the maximum moment region for several load cases: three-point bending, four-point bending and uniformly distributed bending by adopting linear bond-slip relationship without softening

behavior (**Figure 6a**) for modeling interfacial behavior. It was concluded that high interfacial shear stresses at the ends of cracks are mainly responsible for debonding of FRP in cracked R/C beam strengthened with FRP and the gradient of the moment, or shear force has an insignificant effect on the interfacial shear stress distribution. By ignoring the minor terms in the derived expressions for interfacial shear stress the maximum shear stress  $\tau_{max}$  in all load cases converges to

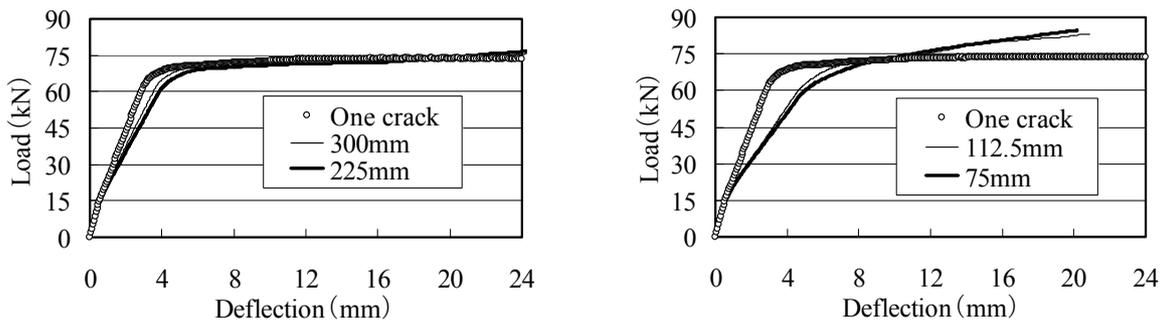
$$\tau_{max} = \frac{f}{b_1} \sqrt{\frac{k_s}{E_1 t_1}} \quad (2)$$

where  $f$  is the axial force in FRP at crack;  $k_s$  is shear stiffness of employed interfacial bond-slip curve;  $E_1$ ,  $t_1$  and  $b_1$  are elastic modulus, thickness and width of FRP, respectively.

Using the concept of fracture energy, the above equation can be rewritten in the same form as **Equation (1)** derived from simple shear test:

$$f_{max} = b_1 \sqrt{2G_f E_1 t_1} \quad (3)$$

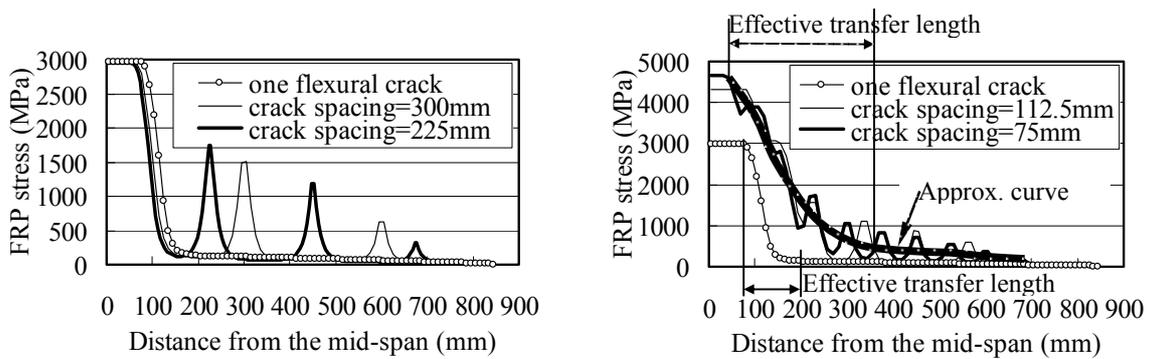
where  $f_{max}$  is the maximum transferable force in FRP at crack in FRP-strengthened beam.



(a) Single Crack and Large Crack Spacings

(b) Short Crack Spacings

**Figure 7.** Load versus Deflection Curves



(a) Single Crack and Large Crack Spacings

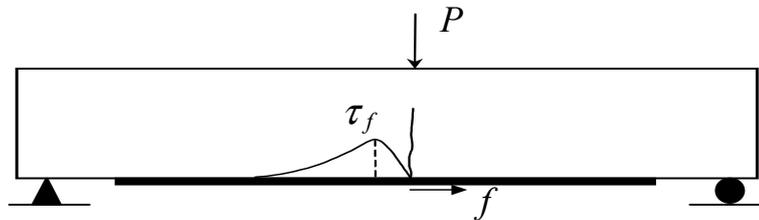
(b) Short Crack Spacings

**Figure 8.** FRP Stress Distributions

In view of the fact that interfacial shear transfer behavior can be well represented by bilinear bond-slip relationship (**Figure 6b**) with consideration of softening behavior [23], Niu and Wu [15] presented a closed-form analytical solution for predicting interfacial shear stress and FRP stress distributions caused by flexural cracks in FRP-strengthened R/C beam, and clarified the debonding mechanism caused by one flexural crack using bilinear interfacial model. Provided that FRP force at end of flexural crack in strengthened beam is same as that at load end in pull-push shear test, it is found that interfacial shear stress distributions are almost same for both FRP-strengthened beam and pull-push shear specimen [18]. For the case of single crack or localized one in FRP-strengthened plain beam, FRP stress keeps constant and load increases no more once debonding is initiated and propagated outwards to end of FRP. Considering that theoretical study is incapable to present the debonding propagation and stress redistribution between cracks, Niu and Wu [16] conducted a finite element analysis and found that crack spacing has a significant effect on debonding failure mechanism and ultimate load-carrying capacity by employing discrete crack model to model concrete crack propagation and bilinear bond-slip relationship to model interfacial behavior. Large crack spacing, or rather crack spacing larger than effective transfer length, yields almost the same ultimate load as that of one crack case, while short crack spacing causes a different debonding mechanism to that of one crack case and the ultimate load is higher. With respect to short crack spacing, or crack spacing shorter than effective transfer length, debonding initiation does not mean the final debonding failure. Debonding propagation is “resisted” by the adjacent crack and much more energy is needed to make the stress redistribution, which to some degree corresponds to increasing the shear transfer length and thus contributes to sustaining increase in FRP stress and external load. **Figure 7** and **8** demonstrate effect of crack spacing on structural response. It is shown that with decrease of crack spacing FRP can be effectively utilized and the corresponding stress distribution can be approximated to a monotonically smoothly curve similar to that of one crack case, with an increased effective transfer length.

### Analytical Model for Predicting Debonding Failure

In FRP-strengthened R/C beam, intermediate crack induced debonding failure is often accompanied by two crack patterns, i.e. localized and distributed crack patterns. Based on the above discussions, the debonding failure mechanism for the case of localized crack pattern is similar to that for simple shear test and thus the corresponding debonding failure can be predicted by combining the fracture energy based **Equation** (1) for determining the ultimate transferable load in FRP and strain compatibility based prediction method. The prediction procedure can be simplified in **Figure 9**.



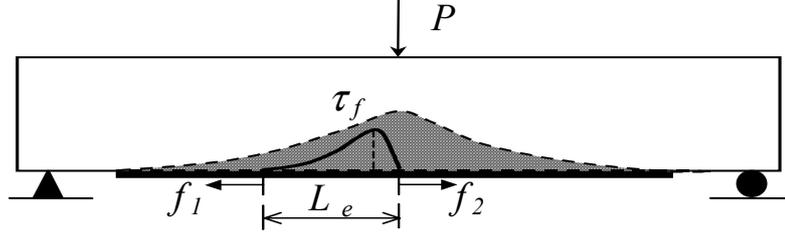
$$(1) P \Rightarrow f$$

$$(2) f \geq b_1 \sqrt{2G_f E_1 t_1} \rightarrow \text{debonding failure}; \quad f < b_1 \sqrt{2G_f E_1 t_1} \rightarrow \text{no debonding}$$

**Figure 9.** Prediction for Debodning Failure Accompanied by Localized Crack Pattern

Yoshizawa and Wu [24] investigated the crack behavior in concrete through conducting uniaxial tension tests and bending test on FRP-strengthened structures with and without reinforcing steel. It is

found that the average crack spacing is averaged from 70 mm to 100mm in FRP-strengthened RC structures. In view of that interfacial shear stress is mainly caused by difference between FRP stresses, for case of debonding failure by distributed cracks, it can be regarded that cracks are uniformly smeared over the whole beam and debonding failure is result in once debonding is initiated between two flexural cracks with a spacing of an effective transfer length. **Figure 8b** shows that this assumption is convincing. Based on such assumption, unified prediction procedure can be established for debonding failure in **Figure 10**, where the case is simplified to that of localized crack if  $f_i=0$  for uncracked section.



- (1)  $L_e = k\sqrt{E_1 t_1}$
- (2)  $P \Rightarrow (f_2 - f_1)$
- (3)  $f_2 - f_1 \geq b_1 \sqrt{2G_f E_1 t_1} \rightarrow \text{debonding failure}; f_2 - f_1 < b_1 \sqrt{2G_f E_1 t_1} \rightarrow \text{no debonding}$

**Figure 10.** Prediction for Debodning Failure Accompanied by Distributed Crack Pattern

According to **Equation (1)**, the effective transfer length  $L_e$  can be approximated by:

$$L_e \approx \frac{2P_{\max}}{\tau_f b_1} = \frac{2\sqrt{2G_f E_1 t_1}}{\tau_f} \propto \sqrt{E_1 t_1} \Rightarrow L_e(\text{mm}) = k\sqrt{E_1 t_1(\text{MPa} \cdot \text{mm})} \quad (4)$$

where  $\tau_f$  is local bond strength of interface,  $k$  is a coefficient determined by experiment, other symbols are defined before.

As stated above, the key parameter in predicting final debonding failure for FRP-strengthened R/C beams is interfacial fracture energy  $G_f$ . In this prediction model, debonding mode, or rather whether debonding occurs in adjacent concrete interface or adhesive, is reflected by the magnitude of  $G_f$ .

## Concluding Remarks

Combining the strain compatibility method and fracture energy-based debonding mechanism, a unified analytical model is established for predicting/evaluating debonding failure caused by intermediate flexural cracks in FRP-strengthened RC beams. It is found that this model is convincing from the theoretical and numerical investigation conducted by the authors. Further investigation, however, is required to be done, such as investigation into the relation between effective transfer length and concrete properties, size effect, and calibration of interfacial fracture energy from a large amount of experimental results.

## References

1. Roberts, T. M. (1989), "Approximate Analysis of Shear and Normal Stress Concentrations in the Adhesive Layer of Plated RC Beams", *The Structural Engineer*, 67(12), 229-233.
2. Ziraba, Y. N., Baluch, M. H., Basunbul, I. A., Sharif, A. M., Azad, A. K., and Al-Sulaimani, G. J. (1994), "Guidelines toward the Design of Reinforced Concrete Beams with External Plates", *ACI Structural Journal*, 91(6), 639-646.
3. Quantrill, R. J., Holloway, L. C., and Thorne, A. M. (1996), "Predictions of the Maximum Plate End Stresses of FRP Strengthened Beams: Part II", *Magazine of Concrete Research*, 48(177), 343-351.
4. Täljsten, B. (1997), "Strengthening of Beams by Plate Bonding", *Journal of Materials in Civil Engineering*, ASCE, 9(4), 206-212.
5. Malek, A. M., Saadatmanesh, H., and Ehsani, M. R. (1998), "Prediction of Failure Load of R/C Beams Strengthened with FRP Plate due to Stress Concentration at the Plate End", *ACI Structural Journal*, 95(1), 142-152.
6. Zhang, S., Raoof, M. and Wood, L. A. (1995), "Prediction of Peeling Failure of Reinforced Concrete Beams with Externally Bonded Steel Plates", *Proceedings of the Institution of Civil Engineers, Structures and Buildings*, 110, 257-268.
7. Al-Sulaimani, G. J., Sharif, A., Basunbul, I. A., Baluch, M. H., and Ghaleb, B. N. (1994), "Shear Repair for Reinforced Concrete by Fiber Glass Plate Bonding", *ACI Structural Journal*, 91(3), 458-464.
8. Khalifa, A., Gold, W. J., Nanni, A., and Abdel Aziz, M. I. (1998), "Contribution of Externally Bonded FRP to Shear Capacity of Flexural Members", *Journal of Composites for Construction*, ASCE, 2(4), 195-203.
9. Triantafillou, T. C. and Antonopoulos, C. P. (2000), "Design of Concrete Flexural Members Strengthened in Shear with FRP", *Journal of Composites for Construction*, ASCE, 4(4), 198-205.
10. Wu, Z. S., Matsuzaki, T., and Tanabe, K. (1997), "Interface Crack Propagation in FRP Strengthened Concrete Structures", *Proceedings of 3rd International Symposium on FRPRCS*, Sapporo, Japan, 319-326.
11. Triantafillou, T. C. (1998), "Fracture Mechanics Approaches to Concrete Strengthening Using FRP Materials", *Fracture Mechanics of Concrete Structures Proceedings FRAMCOS-3*, AEDIFICATIO Publishers, D-79104 Freiburg, Germany, 1761-1770.
12. Niu, H. D., Wu, Z. S. and Asakura, T. (1999), "A Numerical Analysis on Bonding Mechanism of FRP-Strengthened Concrete Structures Using Nonlinear Fracture Mechanics", *Proceedings of the Japan Concrete Institute*, 21(3), 73-78.
13. Wu, Z. S. and Niu, H. D. (2000), "Shear Transfer along FRP-Concrete Interface in Flexural Members", *Journal of Material, Concrete Structures and Pavements*, JSCE, 49(662), 231-245.
14. Wu, Z. S. and Niu, H. D. (2000), "Study on Debonding Failure Load of RC Beams Strengthened with FRP Sheets", *Journal of Structural Engineering*, JSCE, 46A, 1431-1441.
15. Niu, H. D. and Wu, Z. S. (2001), "Interfacial Debonding Mechanism Influenced by Flexural Cracks in FRP-Strengthened Beams", *Journal of Structural Engineering*, JSCE, 47A, 1277-1288.
16. Niu, H. D. and Wu, Z. S., "Debonding and Fracture Mechanics of FRP-Strengthened R/C Beams Influenced by Flexural Cracks", Submitted to *Journal of Composites for Construction*, ASCE.
17. Hognestad, E. (1951), "A Study of Combined Bending and Axial Load in Reinforced Concrete Members", *University of Illinois Engineering Experimental Station*, Bulletin Series No. 399, 128.

18. Wu, Z. S. and Niu, H. D., "Load-Carrying Capacity Due to Debonding Failure of FRP-Strengthened R/C Beams", Submitted to *Journal of Structural Engineering*, ASCE.
19. Wu, Z. S., Niu, H. D., and Yin, J. (2001), "On Interfacial Fracture in FRP-Strengthened Structures", *Proceedings of International Symposium of Young Scholars on Mechanics and Material Engineering for Science and Experiments*, Changsha, China, 72-80.
20. Täljsten, B. (1996), "Strengthening of Concrete Prisms Using the Plate-Bonding Technique", *International Journal of Fracture*, 82, 253-266.
21. Yuan, H., Wu, Z. S., and Yoshizawa, H. (2001), "Theoretical Solutions on Interfacial Stress Transfer of Externally Bonded Steel/Composite Laminates", *Journal of Structural Mechanics and Earthquake Engineering*, JSCE, 18(1), 27-39.
22. Wu, Z. S., Yuan, H., and Niu, H. D. (2002), "Stress Transfer and Fracture Propagation in Different Kinds of Adhesive Joints", *Journal of Engineering Mechanics*, ASCE, 128(3) (in press).
23. Yoshizawa, H., Wu, Z. S. and Yuan, H. (2000), "Study on FRP-Concrete Interface Bond Performance", *Journal of Material, Concrete Structures and Pavements*, JSCE, 49(662), 105-119 (in Japanese).
24. Yoshizawa, H. and Wu, Z. S. (1999), "Cracking Behavior of Plain Concrete and Reinforced Concrete Members Strengthened with Carbon Fiber Sheets", *Fourth international symposium on fiber reinforced polymer reinforcement for reinforced concrete structures*, ACI International SP-188, 767-779.